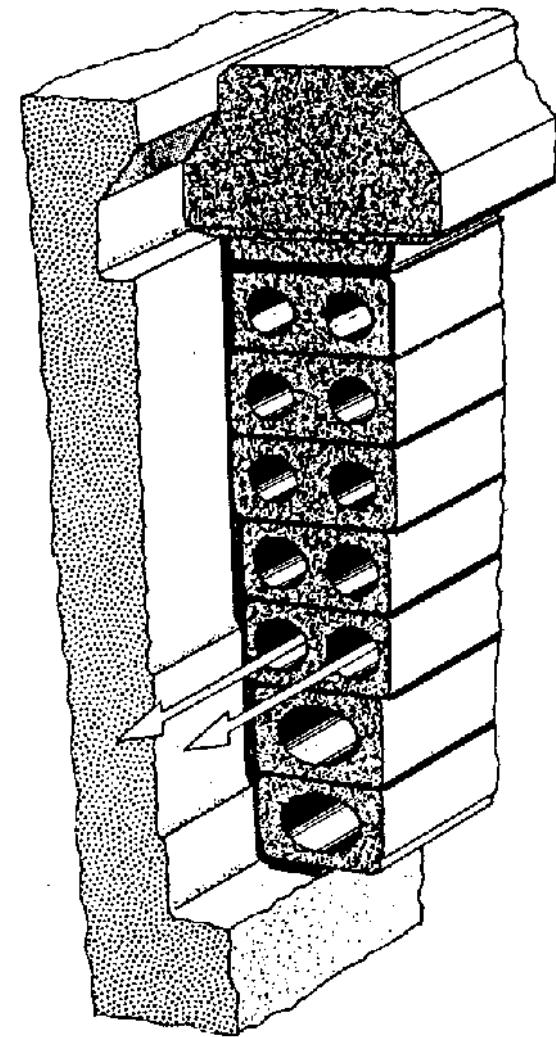


2. Heating and cooling of electrical machines

- Introduction to heating & cooling
- Temperature limits
- Heat sources and loss densities
- Cooling systems
- Coolants
- Basics in fluid dynamics
- Windage losses
- Heat transport by coolant
- Heat transfer
- Conduction of heat
- Efficiency of cooling systems
- Transient heat flow



2. Heating and cooling of electrical machines

2.1 Introduction to heating & cooling

Montsinger's rule

Montsinger's rule for transformer oil and solid insulation materials:

Insulation life span L decreases by 50% (taken as average of a large number of tested specimen) with increase of temperature ϑ by 10 K.

$$L(\vartheta + 10K) = 0.5 \cdot L(\vartheta)$$

Example:

Insulation material for Thermal Class F: $L(\vartheta = 155^\circ\text{C}) = 100000$ hours \Rightarrow
 $L(\vartheta = 165^\circ\text{C}) = 50000$ hours

"**KELVIN-temperature**" T ; unit K (basic SI-unit)

CELSIUS-Temperature ϑ ; unit $^\circ\text{C}$

$$T = \vartheta + 273.15$$

Temperature rise: $\Delta\vartheta = \vartheta_2 - \vartheta_1$; unit K



2. Heating and cooling of electrical machines

2.1 Introduction to heating & cooling

High voltage winding insulation based on Mica

- **Mica** has rather temperature-**independent** break-down field strength E_D and permittivity ϵ
- Small arcing (**“partial discharge”** = **“corona”**) in the small air gaps between the insulation layers
- Partial discharge inception voltage **decreases** with **increasing** temperature
- Semi-conducting **“anti-corona”** screen between insulation layers bridges the air gap to avoid partial discharge
- Mica splittings ($A > 1 \text{ cm}^2$) versus Mica flakes ($(A < 1\text{cm}^2)$)
- Permittivity of resin increases with temperature → **Mica splitting**: E in resin decreases ... Partial discharge inception voltage **INcreases** with **increasing** temperature



2. Heating and cooling of electrical machines

2.2 Temperature limits

Thermal Class	INS.: Insulation material / IMP.: Impregnation	Temperature limit
A	INS.: cotton, paper, wood, ... IMP.: asphalt, shellac	105 °C
E	INS.: wire resin based on polyvinylacetate or epoxy; Impregnated paper; IMP.: synthetic resin	120 °C
B	INS.: glass fibre, asbestos, mica ... IMP.: asphalt, shellac, resin varnish	130 °C
F	INS.: as B, IMP.: Epoxy-resin	155 °C
H	INS.: as B, IMP.: Silicon-resin; silicon-rubber	180 °C
200	INS.: mica, ceramics, glass, quartz ...	240 °C



2. Heating and cooling of electrical machines

Maximum admissible temperature rise $\Delta\vartheta$

Indirect air cooling (IEC 60034-1): Maximum admissible temperature rise $\Delta\vartheta$ (at 40°C ambient temperature = coolant's temperature)

Thermal Class	E	B	F	H
AC winding (e.g. three-phase)	75 K	80 K	100 K*)	125 K
Excitation winding (DC)	75 K	80 K	100 K	125 K
Single layer, non-insulated surface	80K	90 K	110 K	135 K
Coils for cylindrical rotor	-	90 K	110 K	-

*) For rated apparent power less than $S_N = 5 \text{ MVA}$: 105 K

Thermal class	E	B	F	H
Hot spot temperature rise over average temperature rise	5 K	10K	15K	15 K

Average temperature rise: Measured via DC resistance cold and hot: $R_\vartheta = R_{20^\circ C} (1 + \alpha_\vartheta \cdot \Delta\vartheta)$

Hot spot temperature rise: Measured with thermo-couples (e.g. Fe-Constantan)



2. Heating and cooling of electrical machines

Maximum admissible temperature ϑ

Direct cooling (air, hydrogen gas, oil, water,...) (IEC 60034-1): Maximum admissible winding temperature ϑ = maximum temperature of coolant; measured at outlet (where maximum temperature occurs)

Evaporation of water and forming of “resin” in oil must be avoided !

Thermal class	B	F
Inner coolant at outlet of directly cooled active parts: - Gas (air, hydrogen, helium, ...) - water, oil, ...	110 °C 85 °C	130 °C 85 °C
AC winding (e.g. three phase)	120 °C	140 °C
Excitation winding (DC) of cylindrical rotors (increases with the number of cooling sections)	100 ... 115 °C	115 ... 130 °C
Other gas-cooled excitation windings (DC)	130 °C	150 °C



2. Heating and cooling of electrical machines

2.3 Heat sources and loss densities $p_d = P_d/V$ (W/m³)

1. Copper losses and winding eddy-current losses: 0.15...1...5 W/cm³

$$p_d = J^2 / \kappa = P_{Cu} / V$$

Example:

$\vartheta = 120^\circ\text{C}$, $\kappa_{Cu,\vartheta} = 1/1.39 \cdot \kappa_{Cu,20^\circ\text{C}} = 41 \text{ Smm}^2/\text{m}$, $J = 7 \text{ A/mm}^2$

$$p_d = J^2 / \kappa_{Cu,\vartheta} = P_{Cu} / V = 1.19 \text{ W/cm}^3$$

2. Iron losses (hysteresis and eddy current losses): at 50 Hz: 0.03...0.15 W/cm³

Example:

Iron sheet: $v_{10} = 1.7 \text{ W/kg}$ (at 1 T, 50 Hz in EPSTEIN-frame).

In stator teeth: tooth flux density 1.8 T Zahninduktion ($\rho_{Fe} = 7850 \text{ kg/m}^3$): $P_{Fe} = 1.7 \cdot 1.8^2 \cdot 7850 = 43237 \text{ W/m}^3$, $p_d = 0.043 \text{ W/cm}^3$. Increase of losses due to manufacturing & field harmonics: $k_{Vd} = 1.85$:
 $p_d = 0.08 \text{ W/cm}^3$.

3. Additional eddy current losses in conducting parts; friction and windage losses



2. Heating and cooling of electrical machines

2.4 Cooling systems

<i>Open ventilation</i>	<i>Totally enclosed machines – surface cooling</i>	<i>Totally enclosed machines with heat exchanger</i>	<i>Hollow conductor cooling</i>
Coolant air	Coolant air or water jacket	Coolant air Heat exchanger: air-air or air-water	Coolant hydrogen gas, oil or de-ionized water
End shields of machine are open for coolant flow	Increase of machine surface by fins or tubes for air Water jacket cooling	Coolant flow is directed through machine and heat exchanger in closed loop	Pump presses coolant through hollow conductors
Usually up to 500 kW, at higher power acoustic noise is too big	Usually up to 2000 kW	Up to 400 MW ("top air" turbo generators)	Up to biggest machine power (2000 MW)
Often shaft mounted fan	Often shaft mounted fan	Shaft mounted fans, external fans	External pump



2. Heating and cooling of electrical machines

Air cooled machines - coolant is air flow

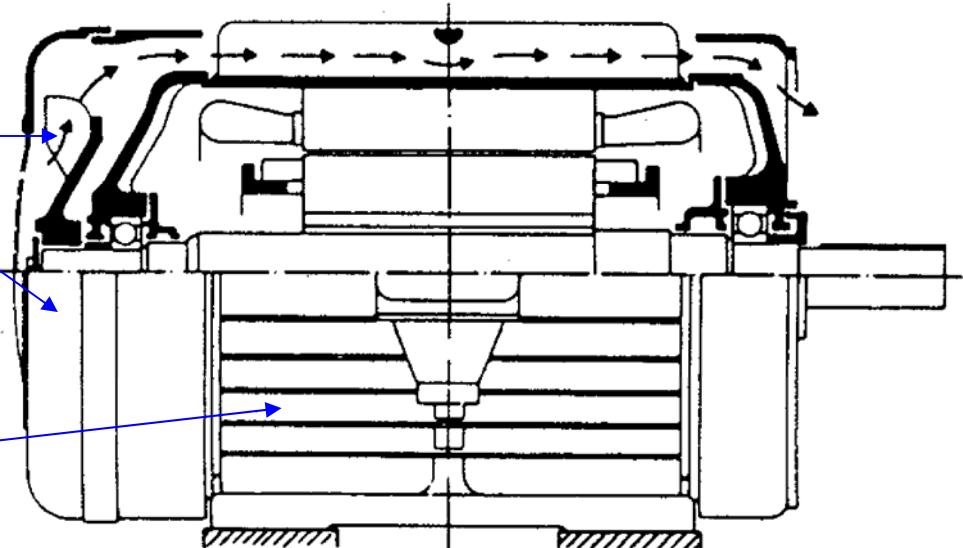
No fan	Shaft mounted fan	Externally driven fan
Cooling only due to natural convection and heat radiation	Speed dependent air flow for cooling	Air flow independent of motor speed
Used for small machines (< 1 kW), e.g. permanent magnet machines due to their lower losses	Used for constant speed drives Big machine power possible	Used for variable speed drives Big machine power possible

Totally enclosed machine – surface cooling

Shaft mounted fan,

fan hood for guiding air flow with air inlet opening,

totally enclosed cage induction machine,
cooling fins on cooling surface



2. Heating and cooling of electrical machines

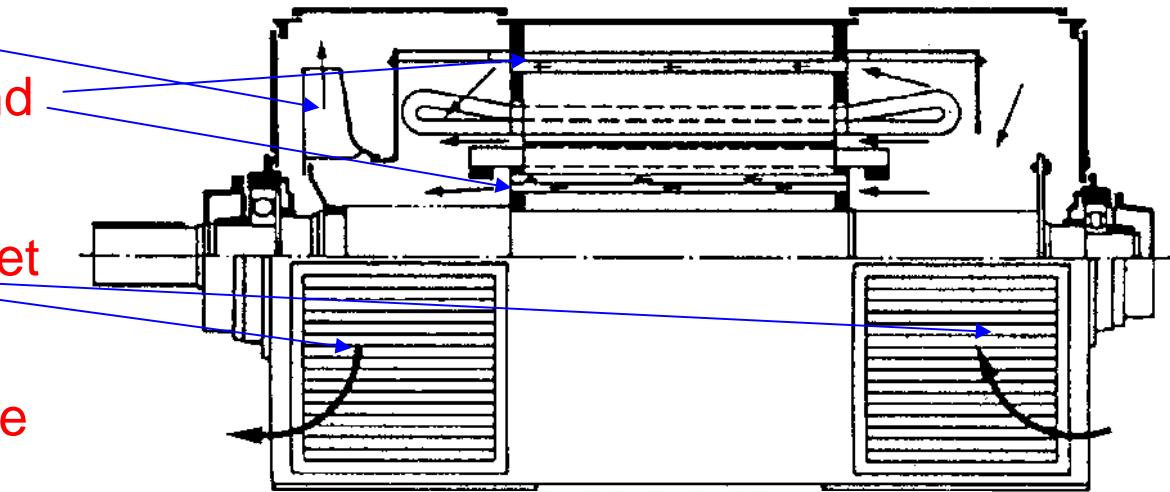
Open ventilation

Shaft mounted fan,

axial ventilation ducts in stator and rotor iron core

openings in end shields for air inlet and outlet,

Example: Cage induction machine



Totally enclosed machines with heat exchanger

Air-air heat exchanger with externally driven fan

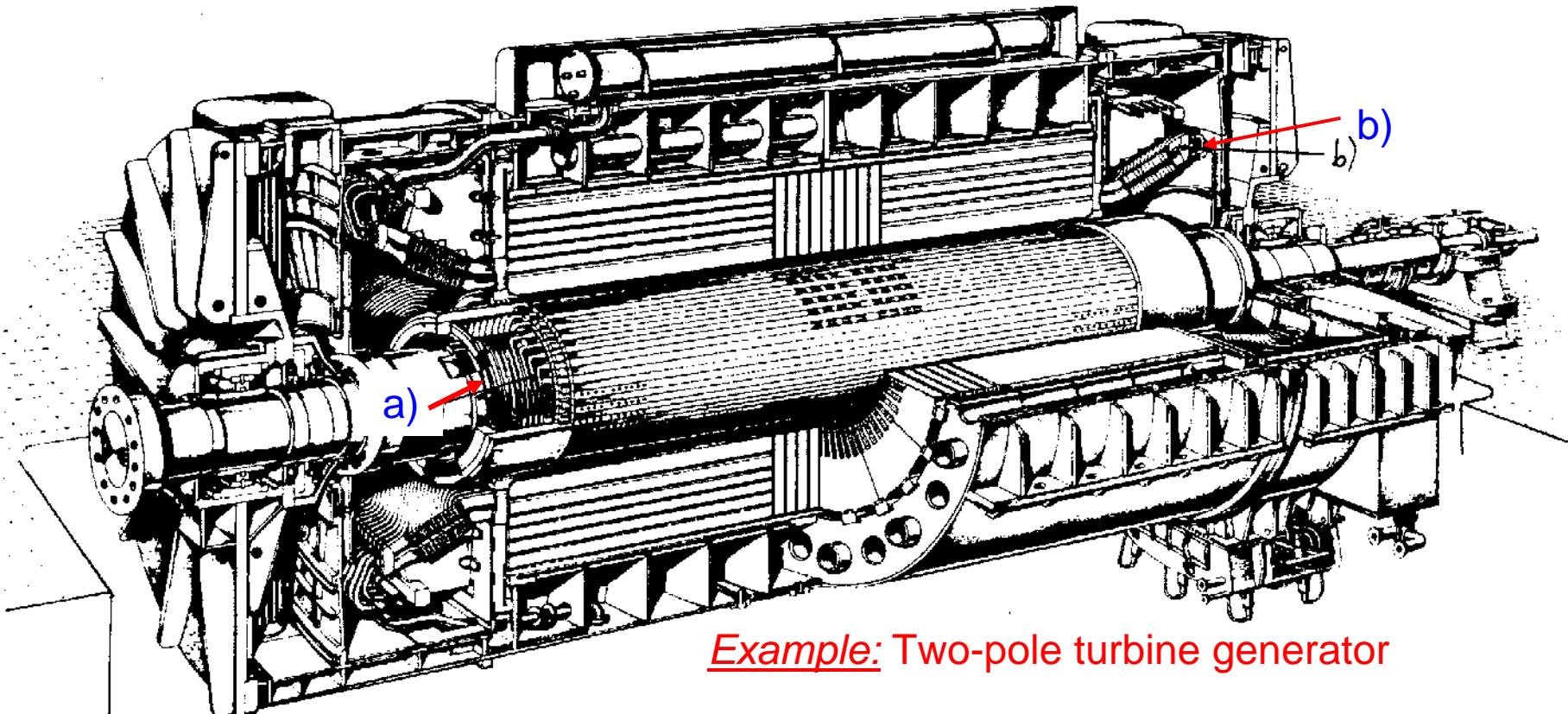
Example:

Wound rotor induction wind generator



2. Heating and cooling of electrical machines

Hollow conductor cooling

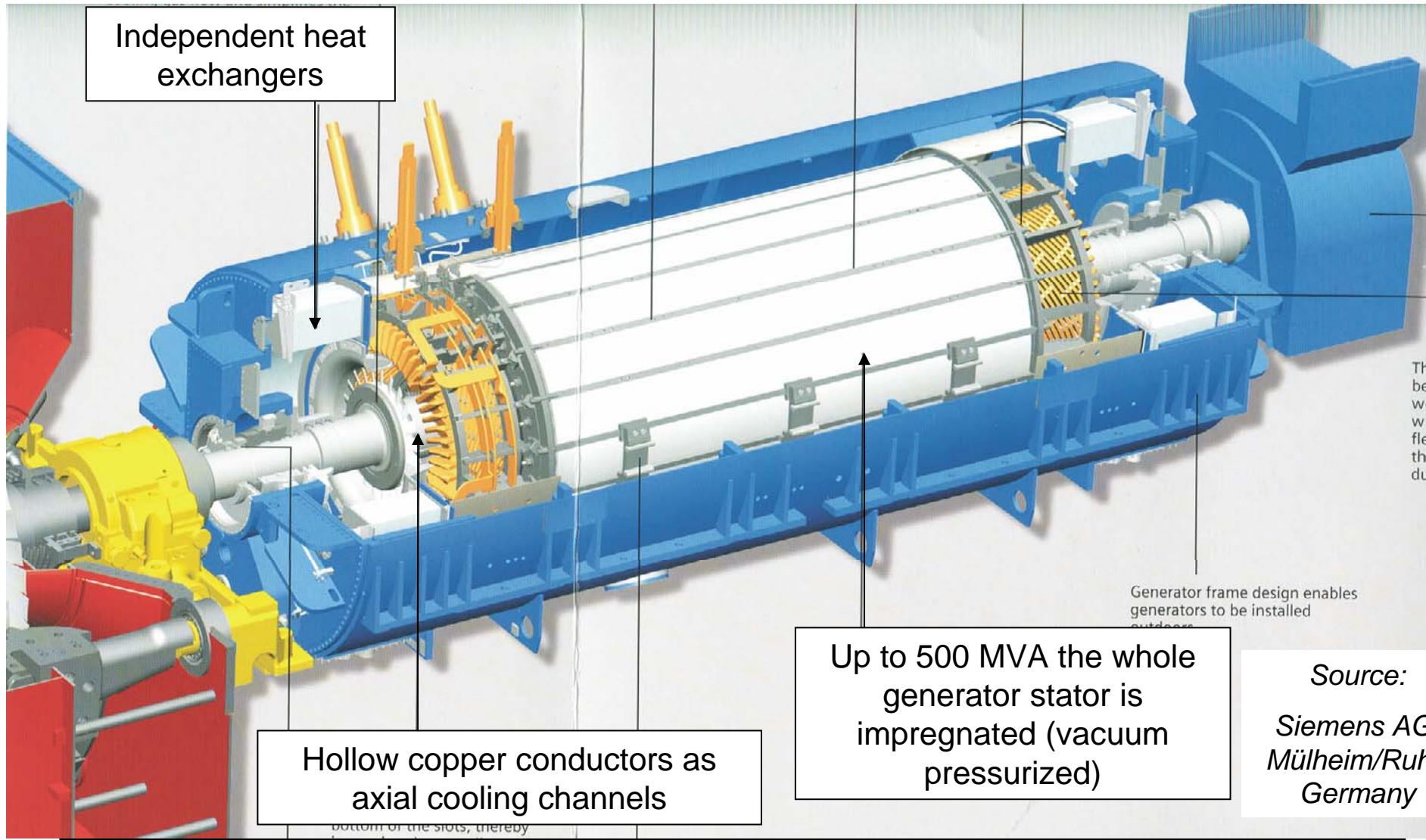


Example: Two-pole turbine generator

- a) Rotor: Direct hydrogen gas cooled hollow copper conductors
- b) Stator: Direct water cooled hollow copper conductors



Testing of a turbine generator in the manufacturers test rig



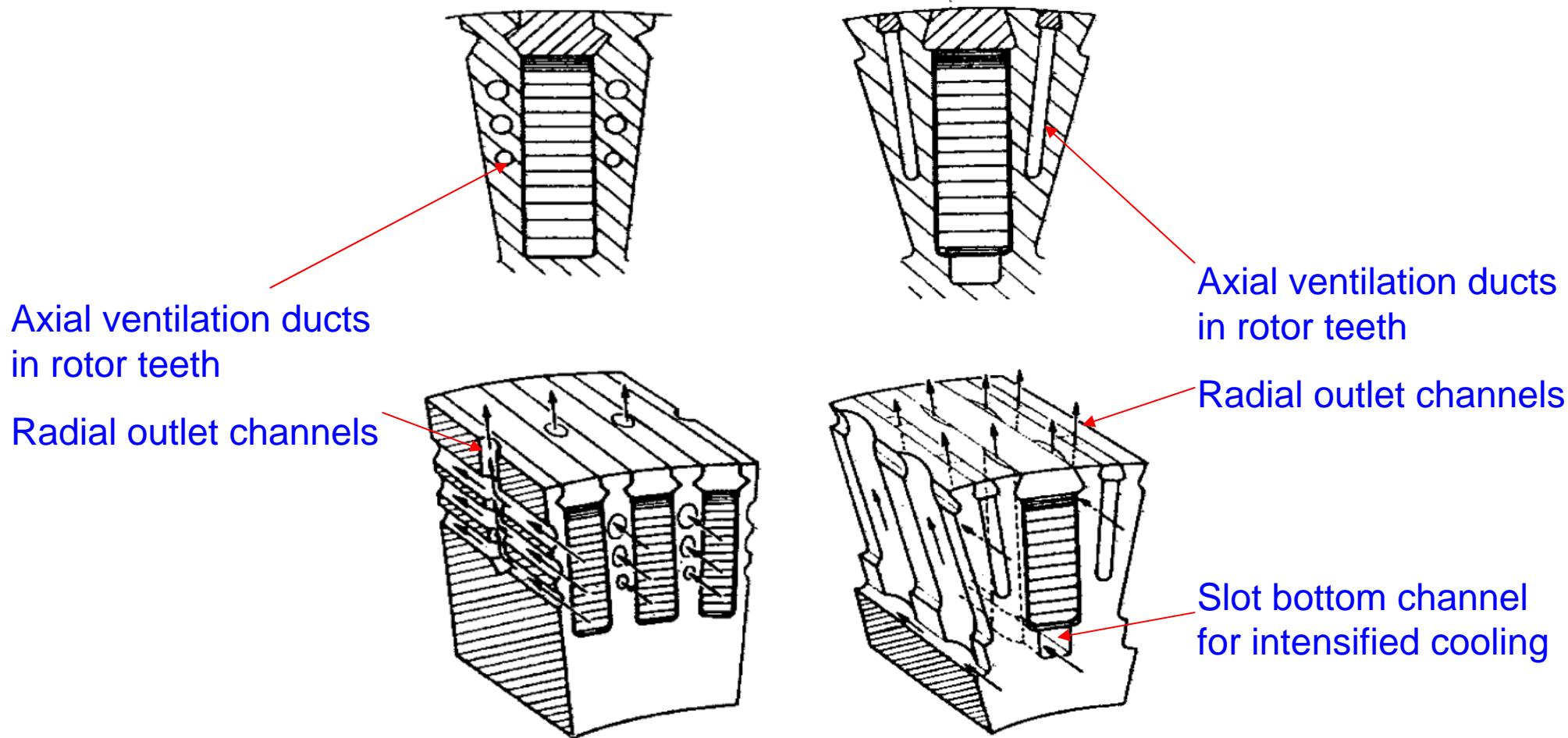
Source:

Siemens AG,
Mülheim/Ruhr,
Germany



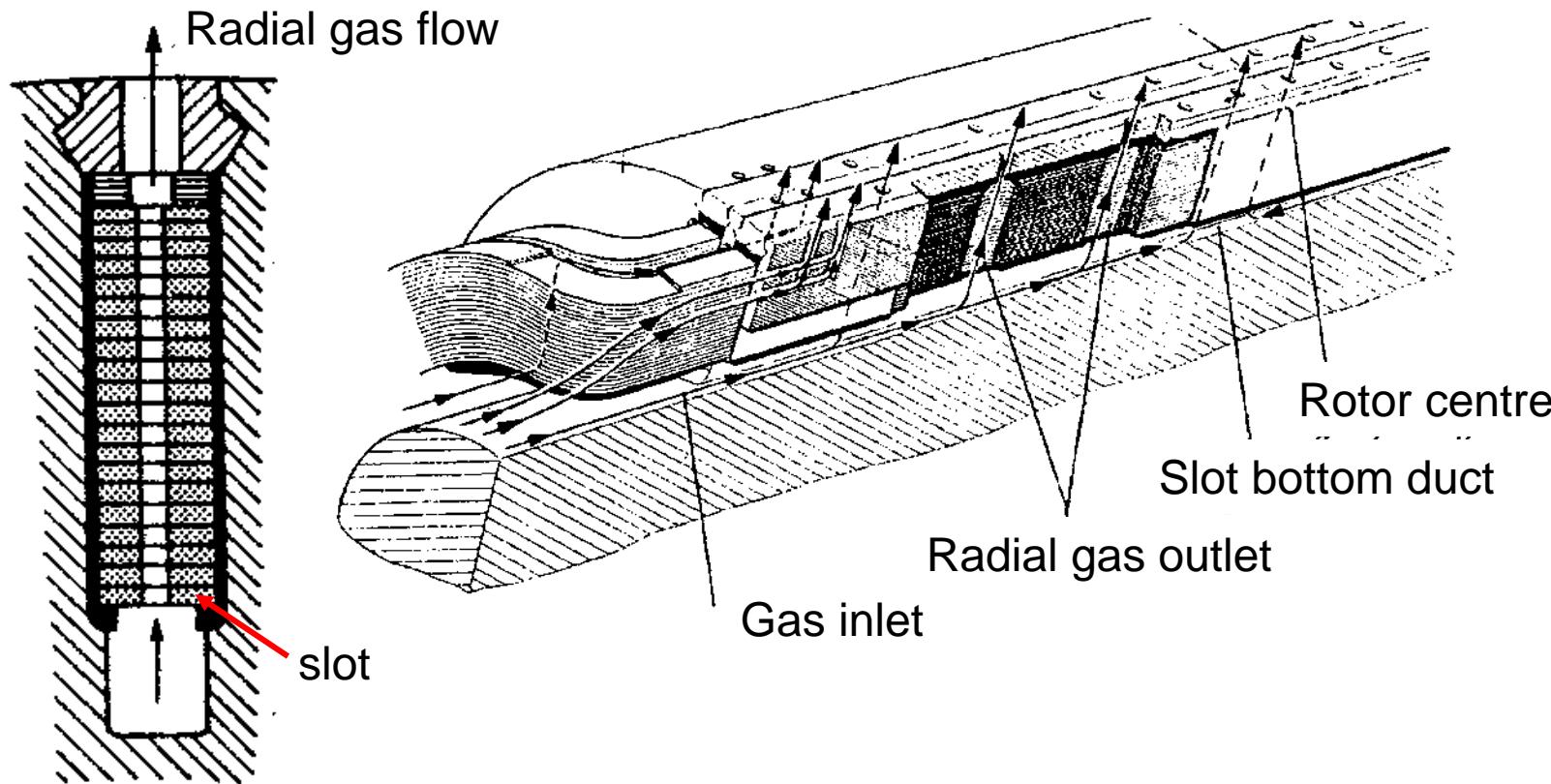
2. Heating and cooling of electrical machines

Indirect gas cooling of turbine generator rotor field winding



2. Heating and cooling of electrical machines

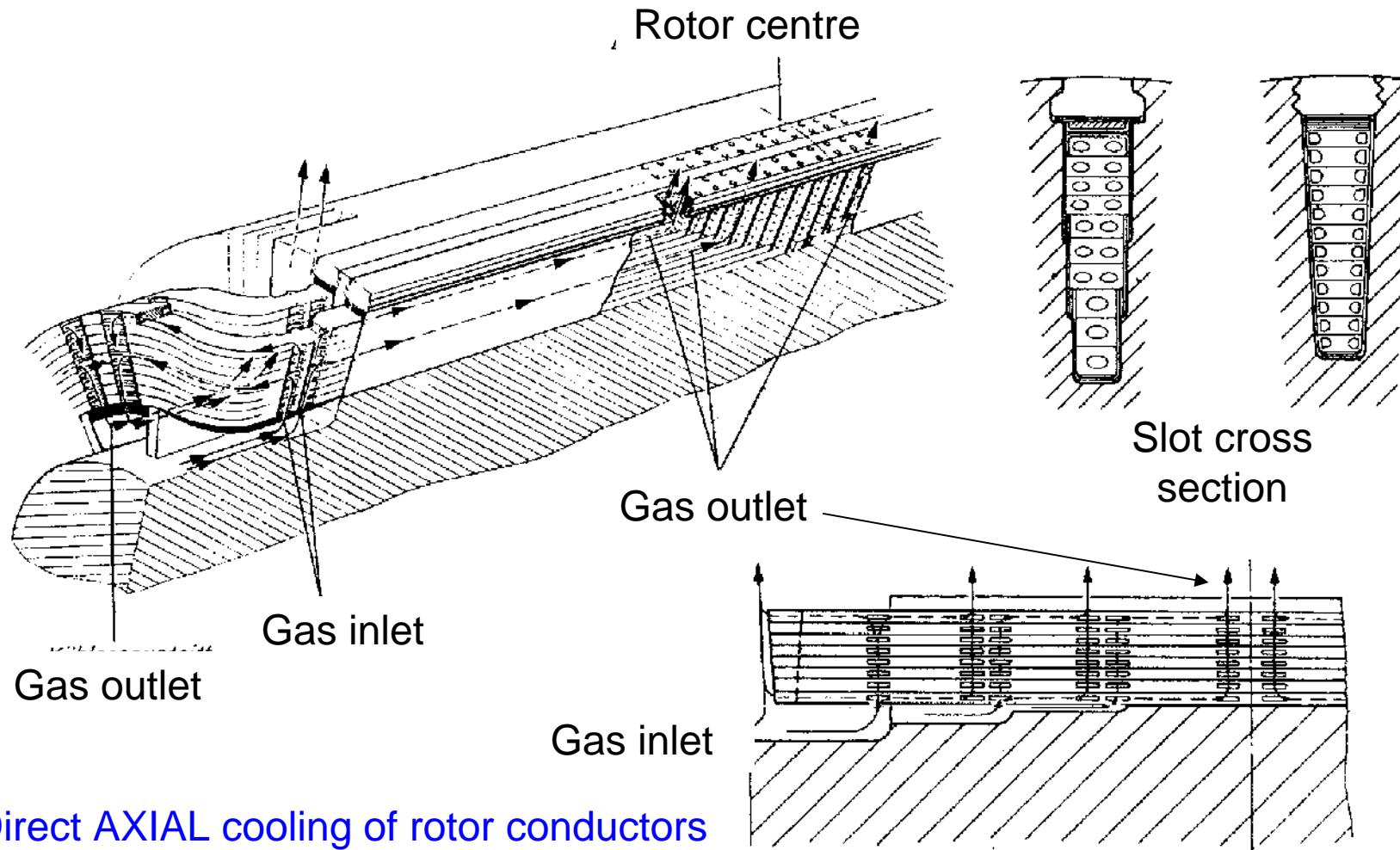
Direct gas cooling of turbine generator rotor field winding (1)



Direct RADIAL cooling of rotor conductors (Air or hydrogen gas)

2. Heating and cooling of electrical machines

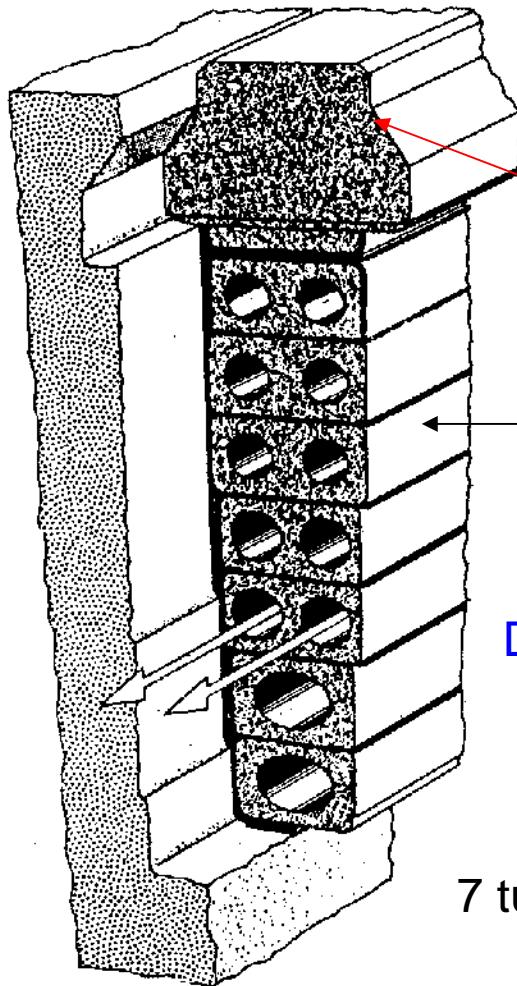
Direct gas cooling of turbine generator rotor field winding (2)



Direct AXIAL cooling of rotor conductors

2. Heating and cooling of electrical machines

Direct gas cooling of turbine generator rotor field winding (3)

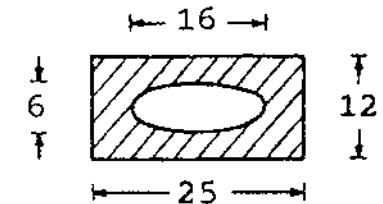
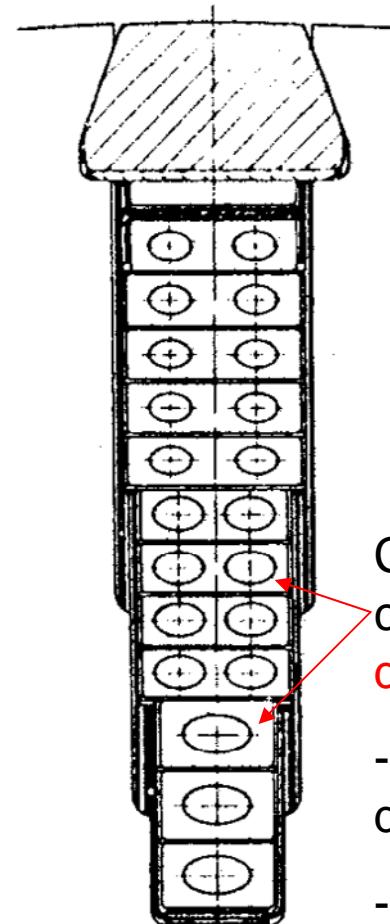


Slot cross section
with
wedge
and
DC conductors in
series

Direct AXIAL cooling of
rotor conductors
(hydrogen gas)

7 turns per slot

12 turns per slot



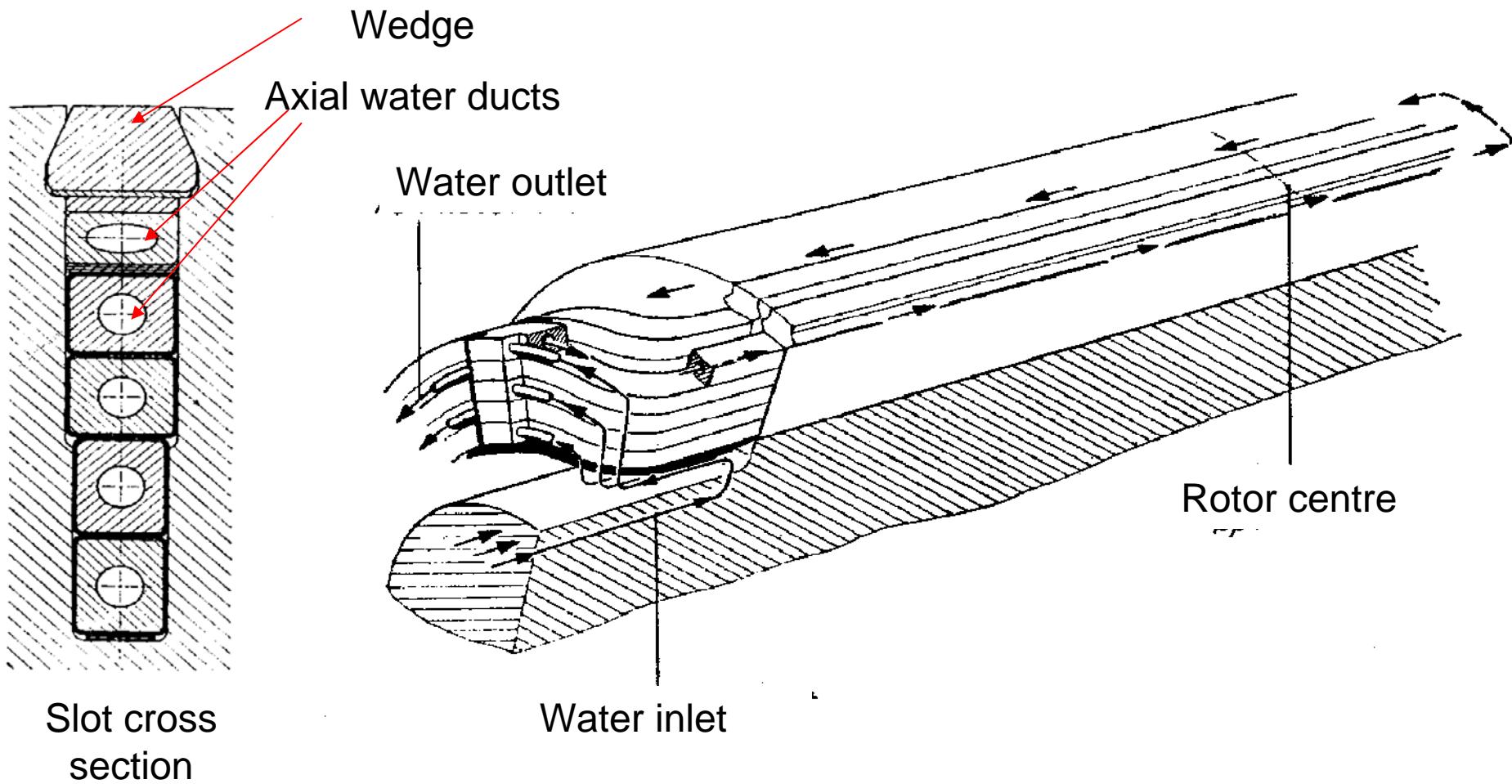
Typical copper
conductor cross
section

- Conductors of **same**
cross section, but
different width allow
- constant current
density
 - constant tooth width



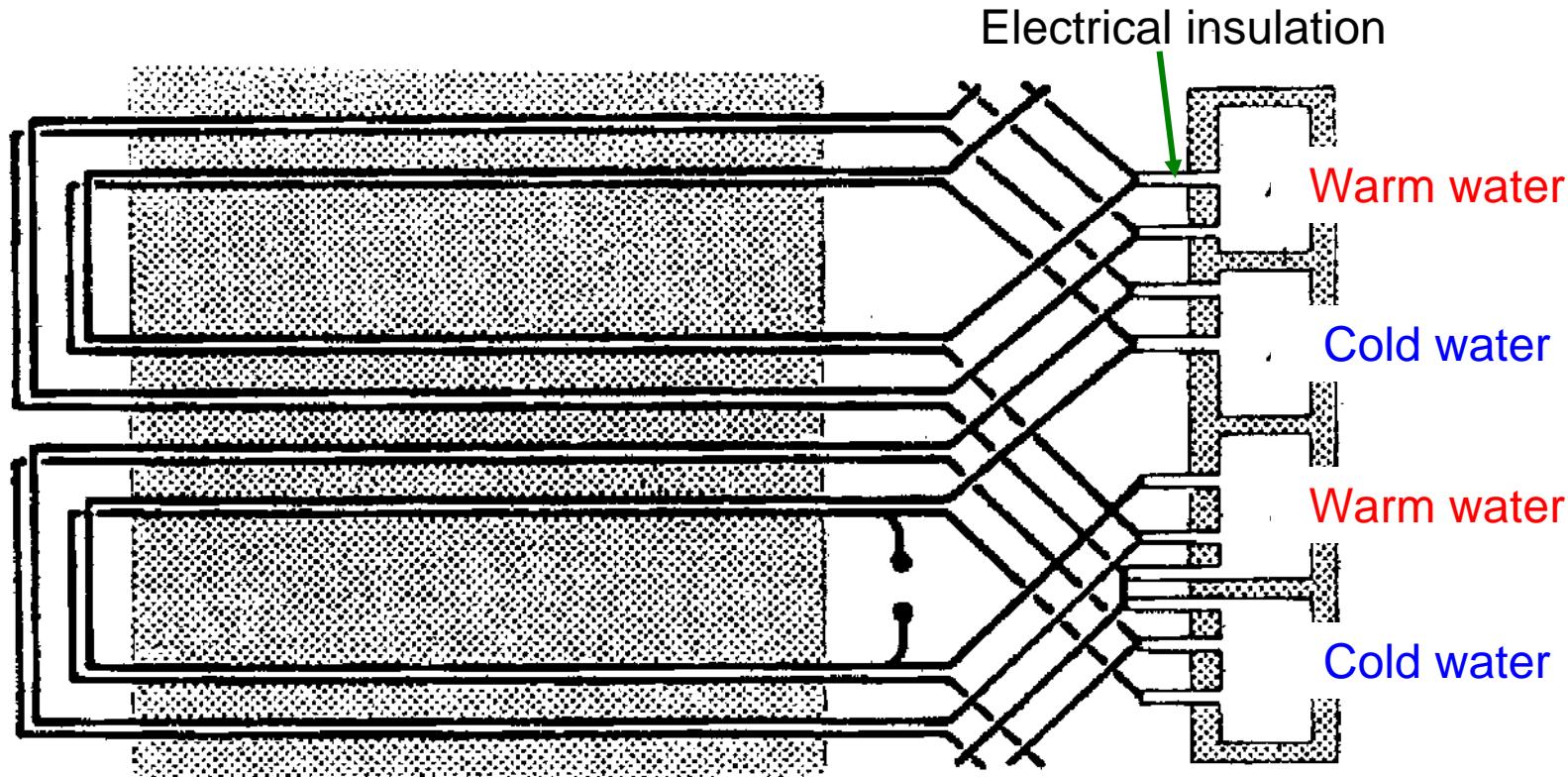
2. Heating and cooling of electrical machines

Direct water cooling of turbine generator rotor field winding (1)



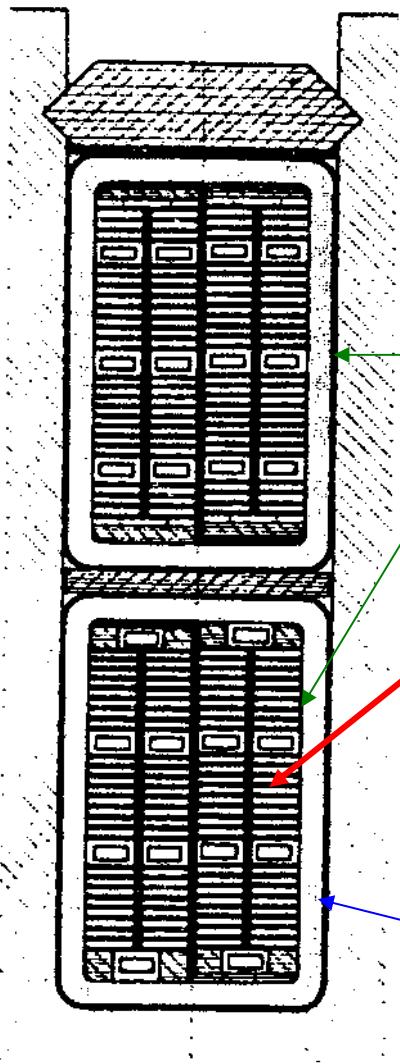
2. Heating and cooling of electrical machines

Direct water cooling of turbine generator rotor field winding (2)



Coils are connected electrically **in series**, but hydraulically **in parallel** to get low pressure drop

2. Heating and cooling of electrical machines



Direct water cooling of turbine generator stator AC winding

Double ROEBEL bar, each made of twisted strands

Two-layer winding

6 out of 54 strands per ROEBEL bar are hollow conductors

They are evenly distributed within the bar !

Hollow conductors are made either of copper or of steel !

2. Heating and cooling of electrical machines

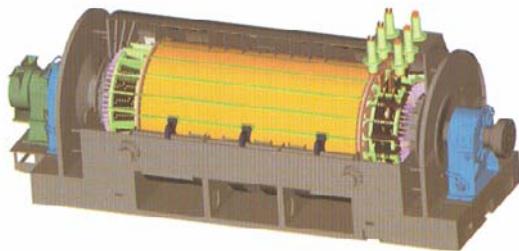
Different cooling systems for different size of generators

Source:

Siemens AG,
Mülheim/Ruhr,
Germany

Air-cooled:

Up to
ca. 350 ... 400 MVA

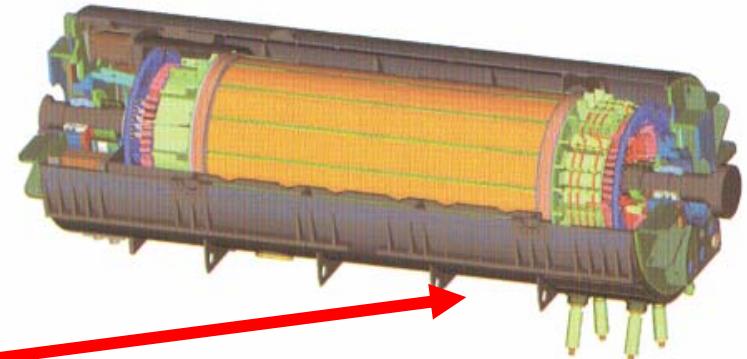


Hydrogen gas-cooled hollow stator and rotor conductors:

Up to ca. 600 MVA

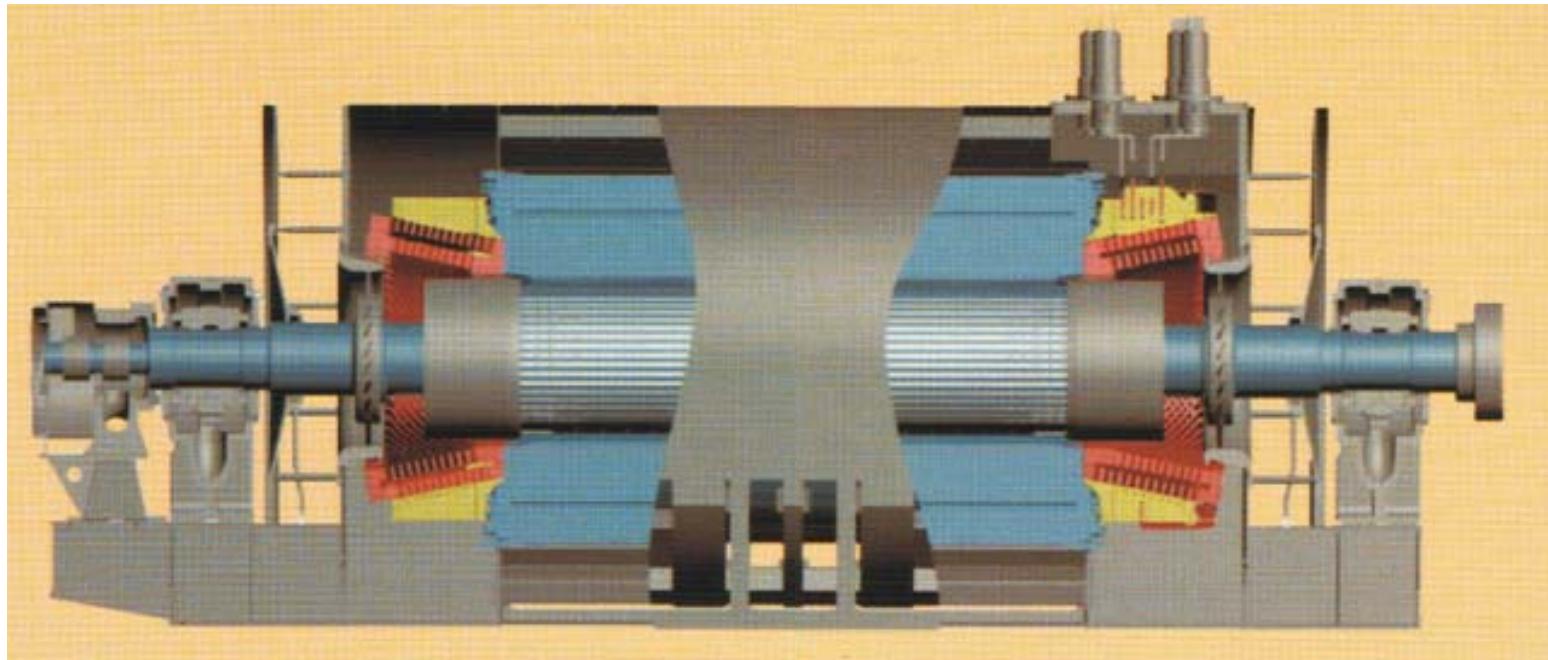


Water-cooled stator hollow conductors, hydrogen gas cooled rotor hollow conductors up to 2000 MVA



2. Heating and cooling of electrical machines

Cross section of air cooled turbine generator



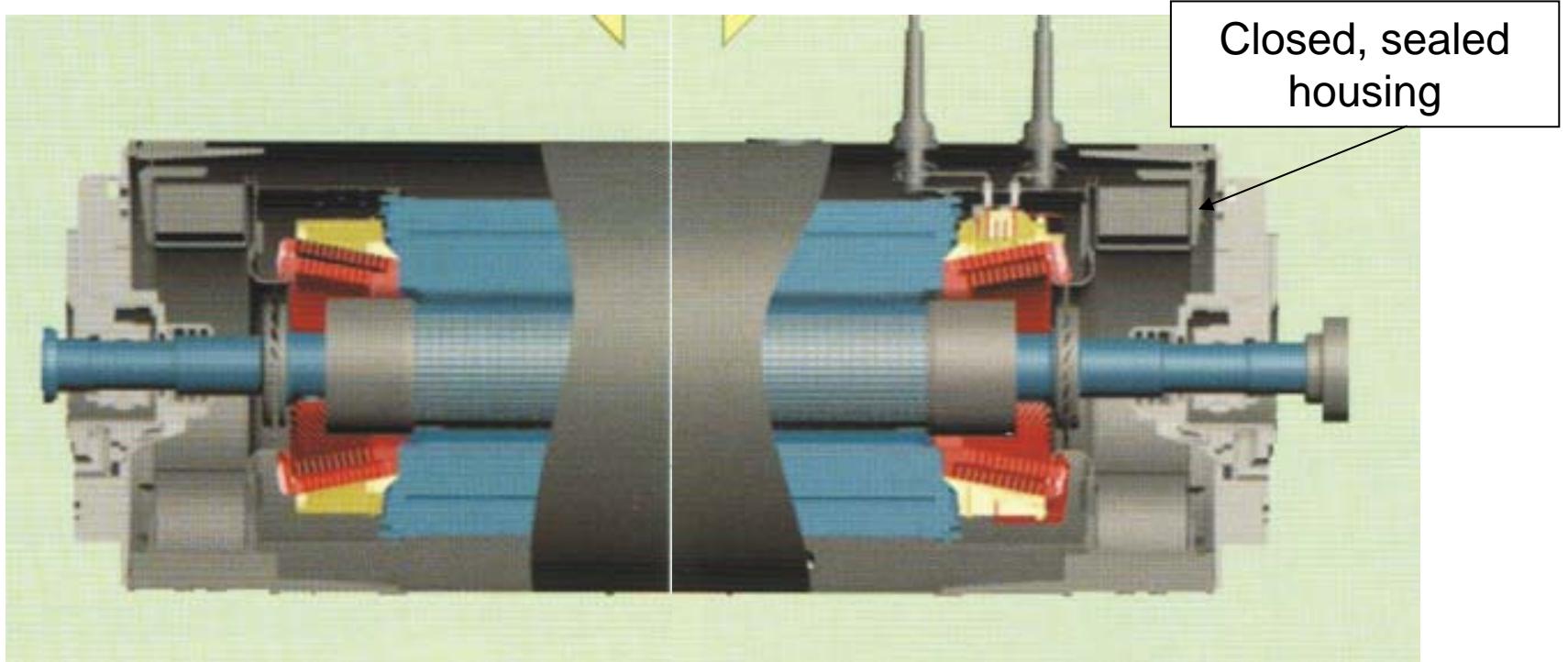
Source:

Siemens AG, Mülheim/Ruhr, Germany



2. Heating and cooling of electrical machines

Cross section of hydrogen gas cooled turbine generator



Source: Siemens AG, Mülheim/Ruhr, Germany



2. Heating and cooling of electrical machines

2.5 Coolants

Properties of gaseous coolants

	Temp- erature ϑ	Mass density ρ	Heat storage capability $c\rho$	Kinematic viscosity ν	Thermal con- ductivity λ	Max. electric field strength E_D
	°C	kg/m ³	Ws/(m ³ K)	m ² /s	W/(m·K)	kV/m
Air	0	1.251	1260	$13.7 \cdot 10^{-6}$	0.024	3200
	50	1.058	1065	$18.4 \cdot 10^{-6}$	0.027	3200
Hydrogen 100 Vol.-%	0	0.087	1236	$98 \cdot 10^{-6}$	0.169	1900
	50	0.0735	1056	$126 \cdot 10^{-6}$	0.183	1900
Hydrogen 96 Vol.-%	0	0.134	1240	$73.4 \cdot 10^{-6}$	0.156	1900
	50	0.113	1060	$94.8 \cdot 10^{-6}$	0.169	1900
Helium He	0	0.173	930	$107 \cdot 10^{-6}$	0.142	1000
Carbon-dioxide CO ₂	0	1.912	1600	$7.2 \cdot 10^{-6}$	0.0143	2900
Nitrogen N ₂	0	1.210	1300	$13.6 \cdot 10^{-6}$	0.0232	3300



2. Heating and cooling of electrical machines

Cooling with air:

Cheap, but must be clean (Filter !). Big velocity necessary for sufficient flow rate, so big aerodynamic noise. Demand for closed ventilation with heat exchangers !

Example:

Turbine generator: 50 MVA, 3000/min, 85 tons total mass;

$\cos\varphi = 0.8$ overexcited, efficiency: $\eta = 97.8\%$

Losses: $P_N = S_N \cos\varphi = P_{out} = 40 \text{ MW}$, $P_d = (1/\eta - 1)P_N = 900 \text{ kW}$,

At 50°C: $\rho_L = 1.058 \text{ kg/m}^3$, $c\rho_L = 1065 \text{ Ws/m}^3\text{K}$,

necessary air flow rate for coolant temperature rise of $\Delta\vartheta = 28 \text{ K}$:

$$\dot{V} = \frac{P_d}{c\rho_L \Delta\vartheta} = \frac{900000}{1065 \cdot 28} = \underline{\underline{30 \text{ m}^3/\text{s}}}$$

- $30 \text{ m}^3/\text{s} = 115 \text{ tons/h} = 1.35\text{-times generator mass per hour} !$
- Closed ventilation necessary !
- Considerable friction losses, so upper machine limit about 400 MVA !



2. Heating and cooling of electrical machines

Cooling with hydrogen gas

Only closed ventilation (danger of explosion, when in contact with oxygen !)

Operation with **higher pressure** than ambient to avoid penetration of oxygen into machine: pressure up to 6 bar.

Increased pressure increases heat transfer capability !

Lower friction losses at higher thermal conductivity than air !

So hydrogen gas is used up to 1000 MVA machine unit power !

No feeding of burning of winding (due to partial discharges) due to lack of O₂.

Danger of explosion with O₂ of air between 4 ... 78 Volume-% H₂ in air !

Optimum for explosion is 30 Vol.-% !

Machines usually are operated with 96 ... 99 Vol.-% H₂ in air !

Sealing of rotating shaft and of housing necessary !

Before filling the machine it is necessary to expel all residual air by inert gas, e.g. carbon dioxide CO₂. Same is done in case of emptying machine e.g. for repair !



2. Heating and cooling of electrical machines

Properties of liquid coolants

	Tem- pera- ture ϑ	Mass density ρ	Heat storage capability $c\rho$	Kinematic viscosity ν	Thermal conductivity λ	Specific electrical resistance $\rho_{el} = 1/\kappa$
	°C	kg/m ³	Ws/(m ³ K)	m ² /s	W/(m·K)	Ωm
Water	20	998	$4174 \cdot 10^3$	$1.01 \cdot 10^{-6}$	0.598	*)
	40	992	$4145 \cdot 10^3$	$0.66 \cdot 10^{-6}$	0.627	
	60	983	$4123 \cdot 10^3$	$0.48 \cdot 10^{-6}$	0.652	
Oil with low viscosity Flash point 120°C	20	800	$1600 \cdot 10^3$	$5.0 \cdot 10^{-6}$	0.147	$10^8 \dots 10^{14}$
	40	785	$1640 \cdot 10^3$	$3.3 \cdot 10^{-6}$	0.143	
	60	770	$1670 \cdot 10^3$	$2.25 \cdot 10^{-6}$	0.140	
Transformer oil Flash point 140°C	20	870	$1760 \cdot 10^3$	$36.5 \cdot 10^{-6}$	0.124	$10^8 \dots 10^{14}$
	40	850	$1820 \cdot 10^3$	$16.7 \cdot 10^{-6}$	0.123	
	60	840	$1860 \cdot 10^3$	$8.7 \cdot 10^{-6}$	0.122	

*) For direct cooling in hollow conductors it is necessary to obtain:
 $(2 \dots 5) \cdot 10^3$ Ohm·m to avoid electric contact to mass potential



2. Heating and cooling of electrical machines

Cooling with de-ionized water

- Water for cooling electrical winding (at high ele. Potential up to 30 kV) must have **low electrical conductivity** ($\kappa \leq 5 \mu\text{S}/\text{cm}$, resp.: $2 \cdot 10^3 \text{ Ohm}\cdot\text{m}$)
- Water has to be also chemically passive, otherwise **corrosion** may happen !
- Water has to be de-mineralized, de-ionized, low content of dissolved oxygen !

In case of **too high water velocity** copper hollow conductors suffer **erosion** !

In some cases also **cavitation** may happen !

Thus velocity v has to be limited:

- **Maximum water velocity v in copper shall be below ca. 2 m/s,**
- in conductors or tubes of non-corrosive steel 3 .. 4 m/s are admissible !



2. Heating and cooling of electrical machines

2.6 Basics in fluid dynamics

Laminar (viscous) and turbulent flow

Flow in tubes :

a) Low velocity: "parallel" orbits of mass particles due to dominating inner viscous forces between particles = **LAMINAR (VISCOSUS)** flow

b) High velocity: Orbita of different particles mingled in "chaotic" way = not only in flow direction, but also perpendicular: **TURBULENT** flow

Based on model parameters: **REYNOLDS number**

$$Re = \frac{v_{av} \cdot d}{\nu}$$

v_{av} : average flow velocity

ν : kinematic viscosity

d : **hydraulic diameter of tube**

$d = 4A/U$ A, U Cross sectional area / circumference of tube

In *straight* tubes with smooth surface:

laminar flow: $Re < Re_{cr}$ (**critical Reynolds number** $Re_{cr} = 2320$)

turbulent flow: $Re > 3000$.

For good heat transfer: Turbulent flow is needed !



2. Heating and cooling of electrical machines

Generation of pressure in fluids by pumps / fans

Radial pump / fan:

Generated pressure due to centrifugal force (= radial force)

F on rotating fluid volume between two blades.

- Rotational speed n , angular frequency $\omega = 2\pi n$), area of cross section between blades A

- Centrifugal force dF on differential small mass element dm at radius r : $dF = \omega^2 \cdot r \cdot dm = \omega^2 \cdot r \cdot \rho \cdot A \cdot dr$

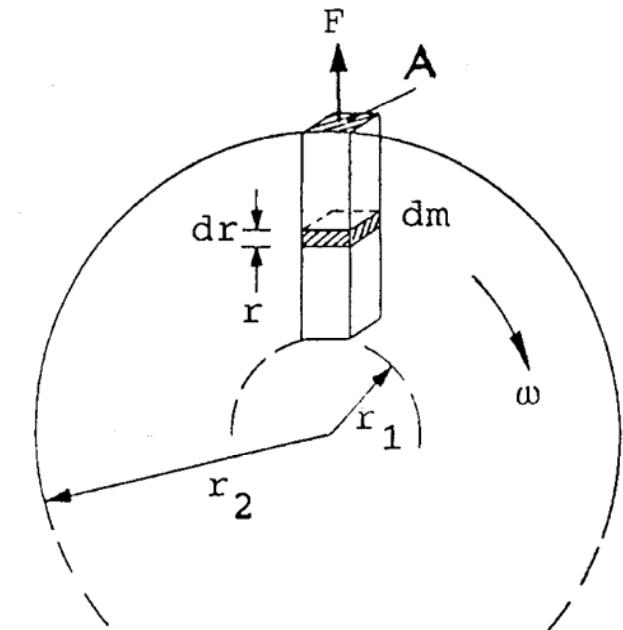
$$F = \int_{r_1}^{r_2} dF = \frac{\rho}{2} \cdot \omega^2 \cdot A \cdot (r_2^2 - r_1^2)$$

Increase of pressure $p = F / A$ between radius r_1 and r_2 :

$$\Delta p_V = \frac{\rho}{2} \cdot \omega^2 \cdot (r_2^2 - r_1^2)$$

Pressure difference increases with

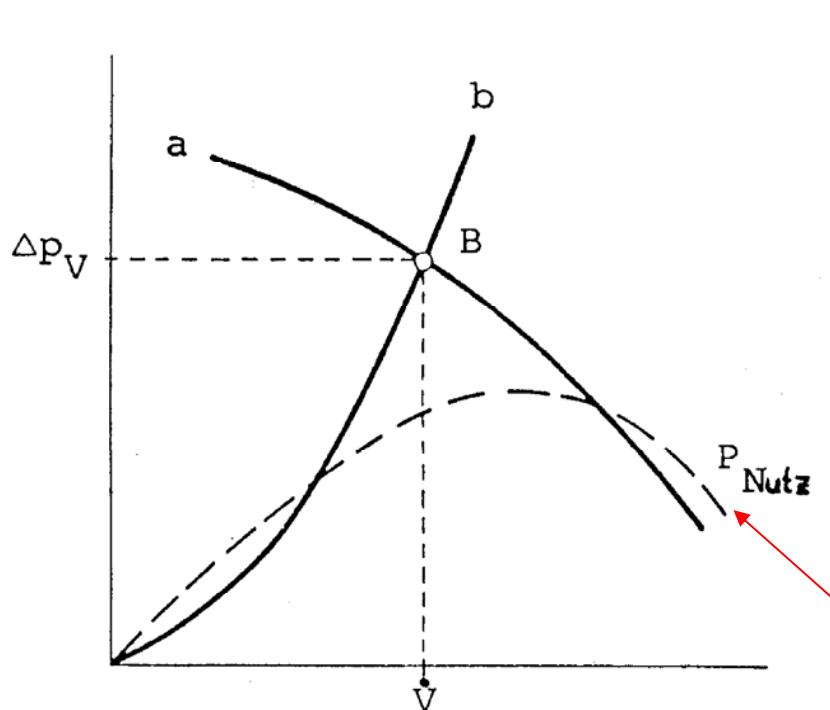
- mass density
- square of speed
- blade length $r_2 - r_1$.



2. Heating and cooling of electrical machines

Pump / fan characteristic

- Flow rate through pump: $\dot{V} = A \cdot v$ A: Total cross section, v: velocity
- Pressure drop in pump due to inner friction rises in turbulent flow with square of speed
- Resulting generated pressure difference hence decreases with increased flow rate:



$$\Delta p_V = f(\dot{V}) = \Delta p_{V0} - k \cdot \dot{V}^2 \quad \Delta p_{V0} = \frac{\rho}{2} \cdot \omega^2 \cdot (r_2^2 - r_1^2)$$

- Pump / fan has to act against **pressure drop of hydraulic system**, which also increases with square of flow rate.

- a) Pump generated pressure difference
 - b) Pressure drop of hydraulic system
- B: Operating point of pump / fan

Pump / fan output power $P_{Nutz} = \dot{V} \cdot \Delta p_V$

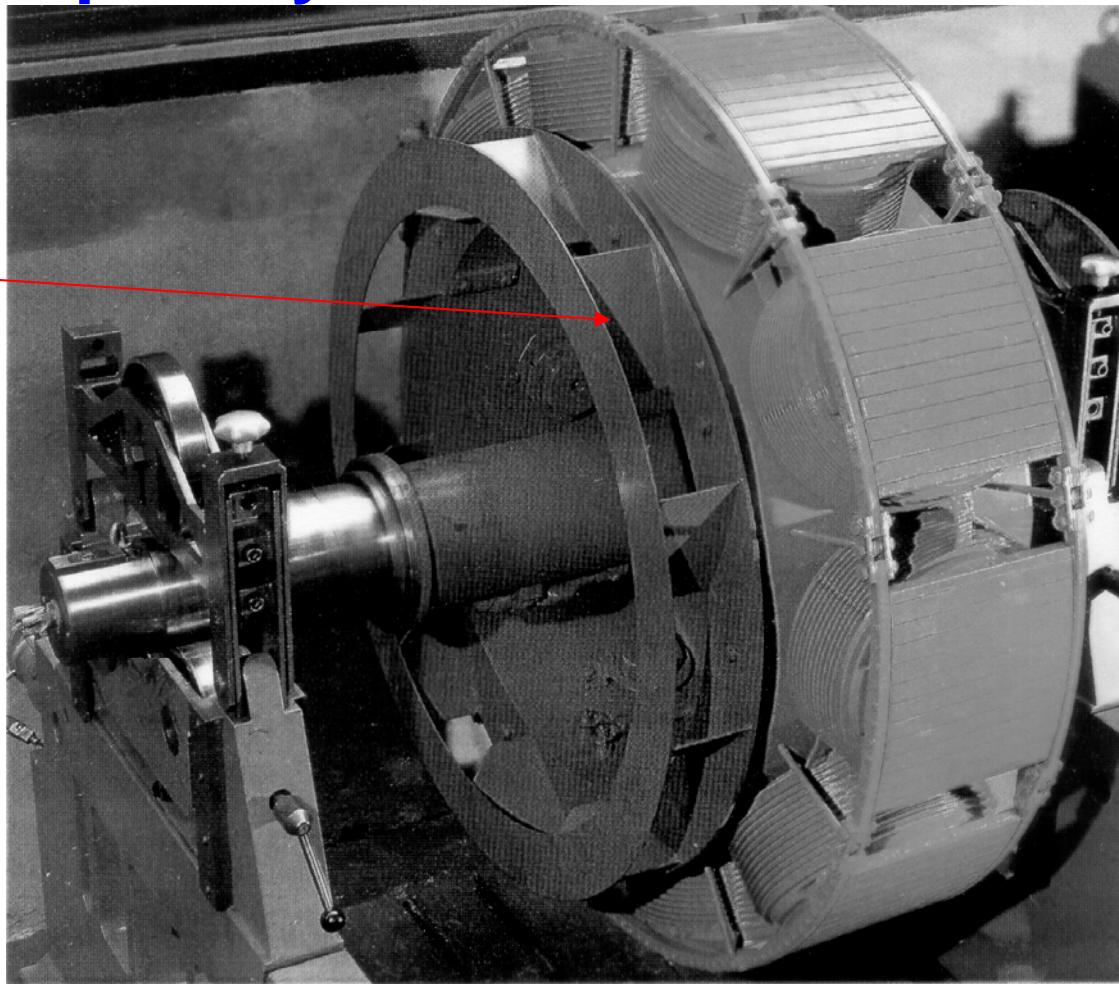
= Power transferred to fluid to move it !



2. Heating and cooling of electrical machines

Radial fan with backward bent blades for a salient pole synchronous machine

Radial fan,
shaft mounted



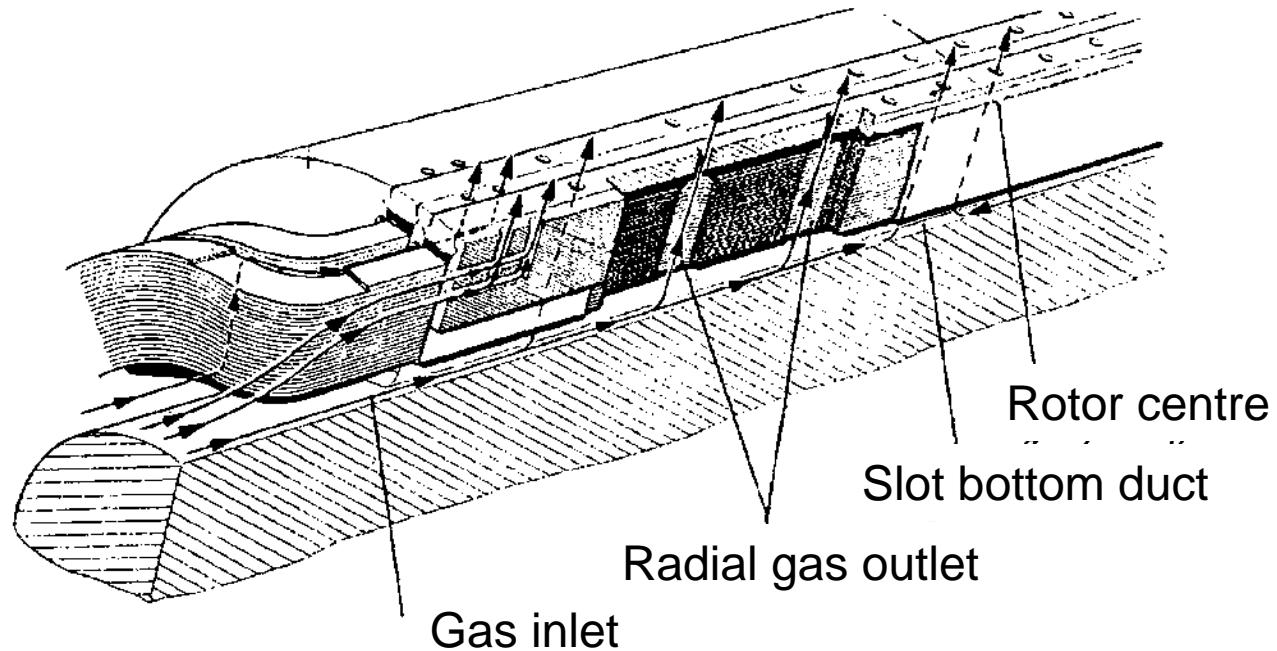
Source:
Siemens AG,
Germany



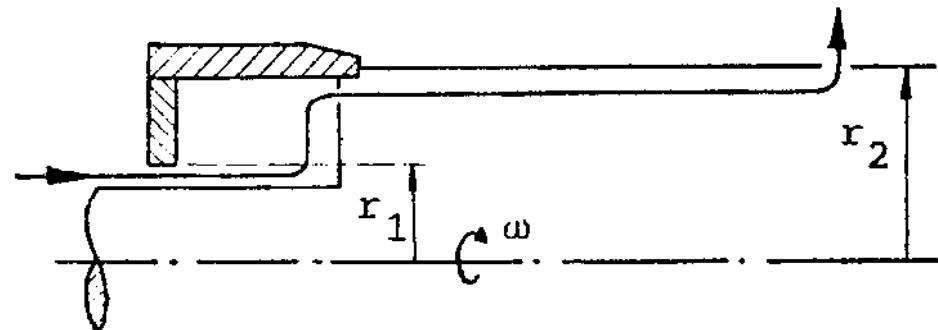
2. Heating and cooling of electrical machines

Generation of pressure in rotor winding

$$\Delta p_V = \frac{\rho}{2} \cdot \omega^2 \cdot (r_2^2 - r_1^2)$$



Direct RADIAL cooling of rotor conductors (Air or hydrogen gas)



2. Heating and cooling of electrical machines

Generation of pressure in axial pumps / fans

- No difference in radii at inlet and outlet of pump / fan = no centrifugal force difference
- Generated pressure due to different speed of flow on both sides of blade.

Bernoulli-equation: Subscript 1, 2: Both sides of axial blade

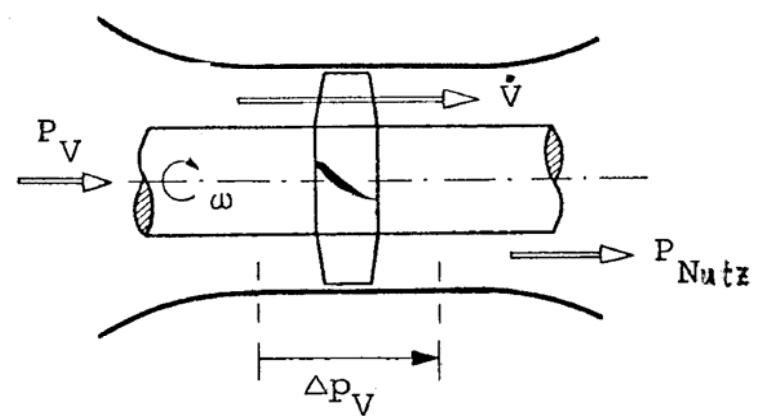
$$\rho \cdot v_1^2 / 2 + p_1 = \rho \cdot v_2^2 / 2 + p_2 \Rightarrow \Delta p = p_2 - p_1 = \rho \cdot (v_1^2 - v_2^2) / 2 \sim \rho \cdot v^2$$

- Circumference speed at radius r is v_u : It is proportional to speed of flow along blades v .

$$v \sim v_u = 2\pi \cdot r \cdot n$$

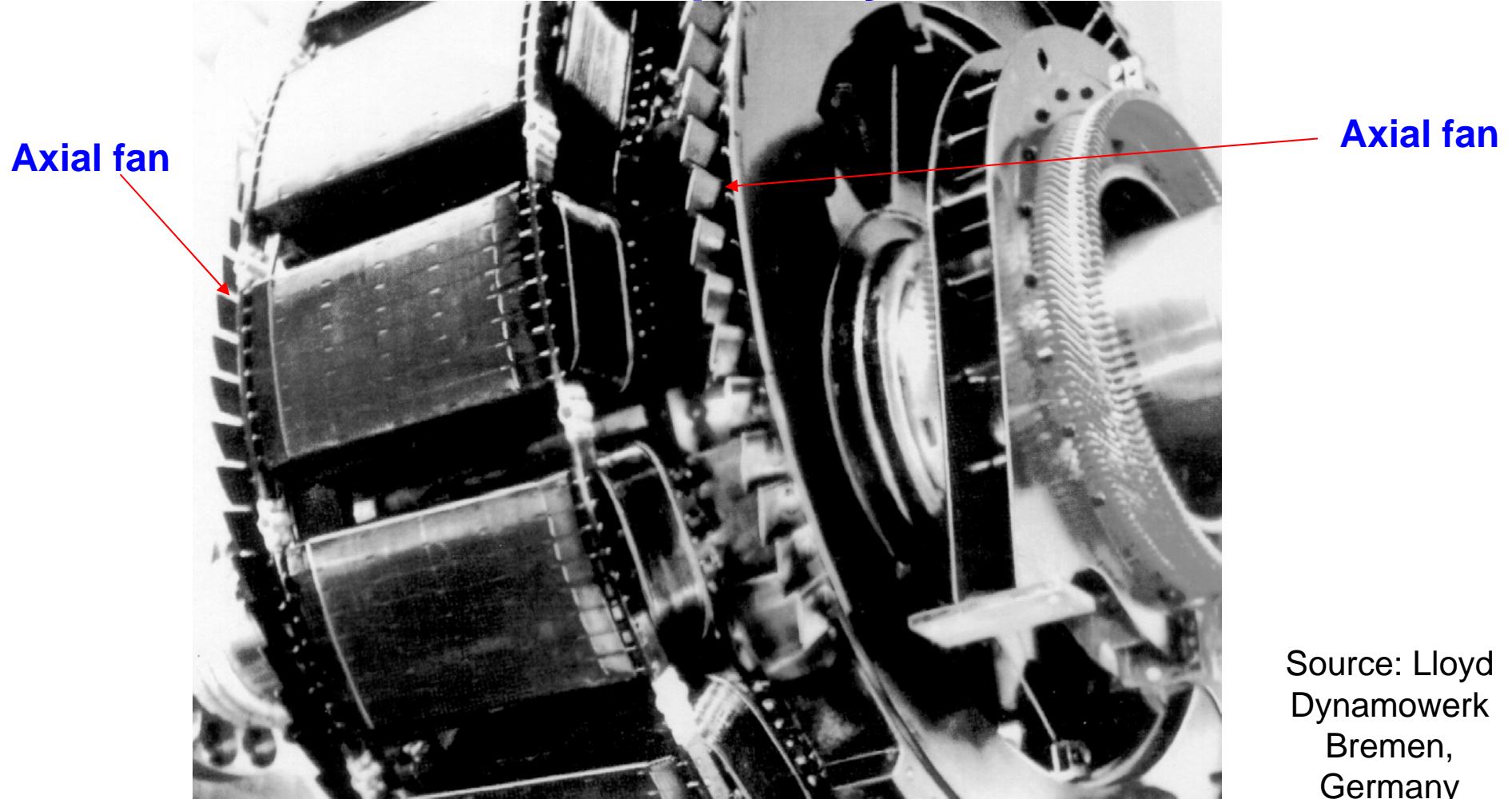
- Pressure difference increases with
- mass density
- square of speed

$$\Delta p_V \sim \rho \cdot n^2$$



2. Heating and cooling of electrical machines

Axial fan for a salient pole synchronous machine



Source: Lloyd
Dynamowerk
Bremen,
Germany



2. Heating and cooling of electrical machines

Efficiency of fans (ventilators) for gaseous fluids

- Necessary mechanical power at the shaft to drive the fan: $P_V = P_{Nutz} / \eta_V$
- Ventilator efficiency:
 - η_V : 0.1 ... 0.3: simple shaft mounted fans in standard machines for mass production
 - 0.3 ... 0.6: typical shaft mounted fans in bigger machines
 - 0.6 ... 0.85: optimum design for fans (e.g. adjustable blade angle in axial fans, special contoured blades like in propellers etc.)

Fan / pump characteristic at variable speed n in turbulent flow:

- Pressure drop in pump / fan & in hydraulic system: $\sim \dot{V}^2$
- Change of flow rate, generated pressure, power transferred to fluid:

$$\dot{V} \sim n$$

$$\Delta p_V \sim n^2$$

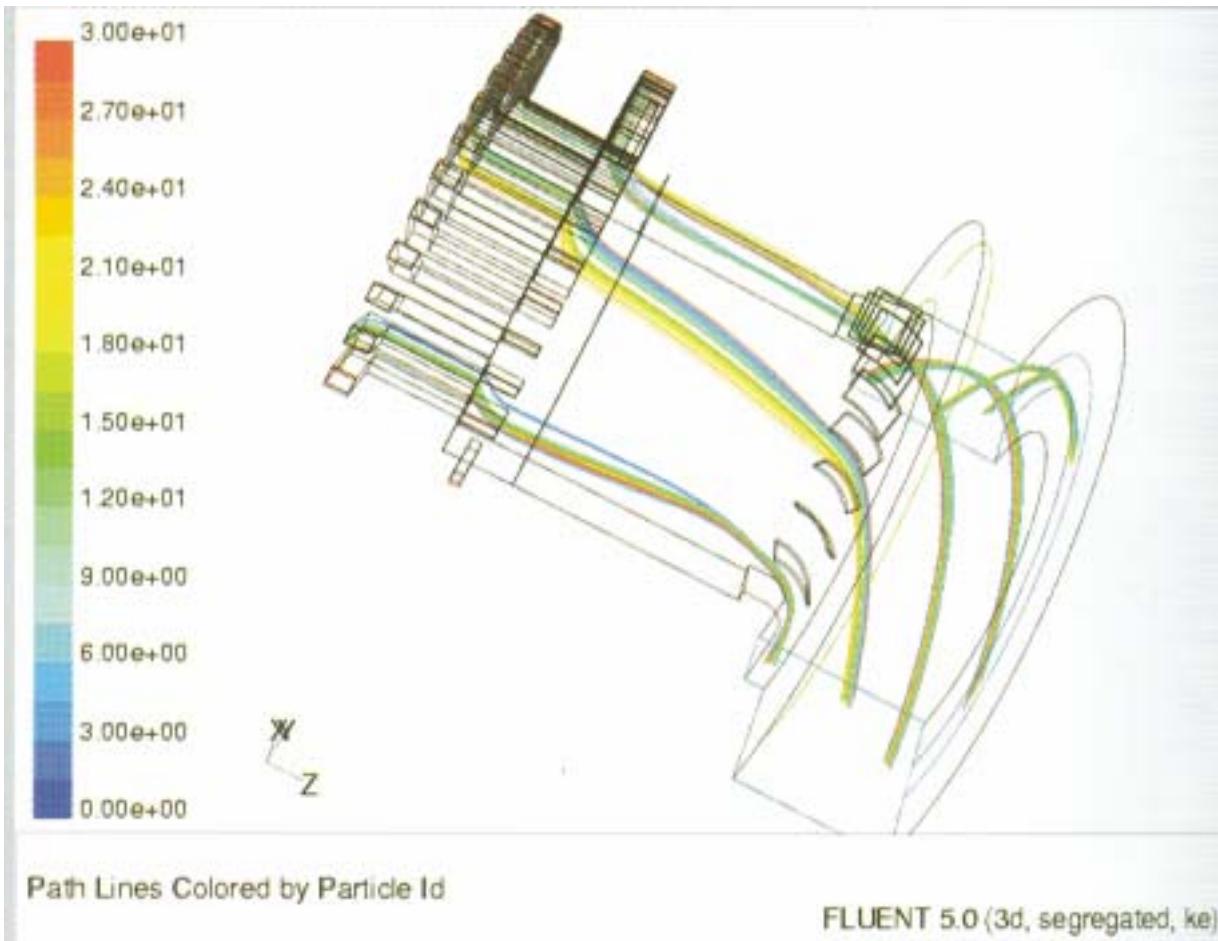
$$P_V \sim n^3$$

(necessary for shaft mounted fan in variable speed drives)



2. Heating and cooling of electrical machines

Computational fluid dynamics



- Simulation of coolant gas particle flow paths via the axial fan
- Air velocity up to 30 m/s
- Left: Outlet from the rotor axial ventilation ducts

Source:

Siemens AG,
Mülheim/Ruhr, Germany



2. Heating and cooling of electrical machines

Pressure drop in hydraulic systems (turbulent flow)

- Inner friction of moved fluid causes **pressure drop**: $\Delta p_H \sim \frac{\rho}{2} \cdot v^2$
- It has to be overcome by pump / fan generated pressure ! Power $P_{Nutz} = \dot{V} \cdot \Delta p_V$ is needed to move fluid against this counter-pressure. Power P_{Nutz} is transferred into heat in fluid.

- Pressure drop **at obstacles** =

= each deviation from smooth, straight tube geometry:

$$\Delta p_H = \zeta \cdot \frac{\rho}{2} \cdot v^2$$

- Outlet from tubes:

= *increasing of cross section of tube*, velocity decreases according to continuity of flow in incompressible fluids: $\dot{V} = A_1 \cdot v_1 = A_2 \cdot v_2$

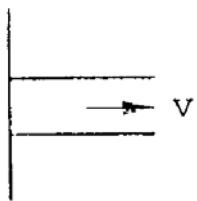
$$A_2 > A_1 : v_2 = \dot{V} / A_2 < v_1 = \dot{V} / A_1$$

If $A_2 \gg A_1$, and sharp edge at outlet: $\zeta \approx 1$ (**CARNOT's law**)



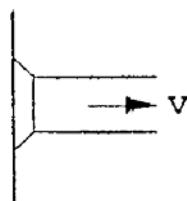
2. Heating and cooling of electrical machines

Pressure drop at obstacles (turbulent flow)



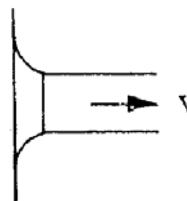
Sharp edge

$$\xi \approx 0,5$$



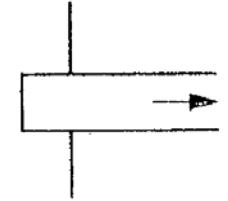
sloped edge

$$\approx 0,25$$



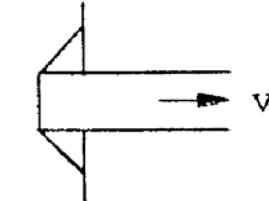
rounded edge

$$\approx 0,06 \dots 0,005$$



projecting

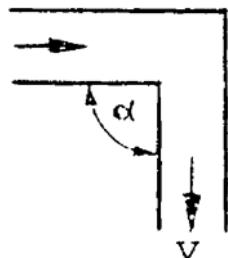
$$\approx 3$$



projecting cone

$$\approx 0,6$$

Inlet into tubes: Shape of tube entrances



für $\alpha = 90^\circ$:

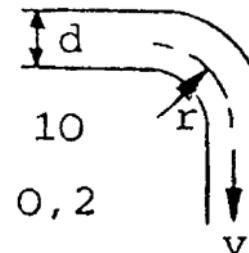
$$\xi \approx 1$$

Curbed tubes:

Sharp curve

für 90° gilt:

r/d	1	2	10
0,51	0,3	0,2	

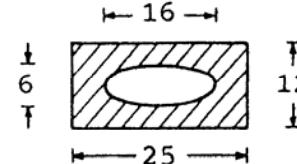


round curve

2. Heating and cooling of electrical machines

Pressure drop in tubes due to friction (turbulent flow)

- Tube with circular or elliptic cross section:



Grade or roughness of inner surface λ_R , length l and hydraulic diameter d determine pressure drop

$$\Delta p_R = \lambda_R \frac{l}{d} \cdot \frac{\rho}{2} \cdot v^2$$

a) Smooth copper tubes: $Re < 10^5$ (**BLASIUS law**): $\lambda_R = 0.316 Re^{-1/4}$

b) Rough surface tubes: (**HOPF & FROMM law**): $\lambda_R = 10^{-2} (k/d)^{0.314}$

k depends on degree of roughness

- New cast iron or iron tubes: $k = 2.5$

- Punched out channels in iron stacks: (e.g. $d = 0.014$ m): $\lambda_R = 0.04$

This corresponds with: $k = 1.15$

$$k = d \cdot \left(\frac{\lambda_R}{10^{-2}} \right)^{\frac{1}{0.314}} = 0.014 \cdot \left(\frac{0.04}{10^{-2}} \right)^{\frac{1}{0.314}} = 1.15$$

2. Heating and cooling of electrical machines

Total pressure drop in hydraulic system (turbulent flow)

- Total pressure drop = sum of all partial pressure drops in hydraulic system !
- Simplest case: One tube with constant cross section (= constant velocity v) and N obstacles:

$$\Delta p_{res} = \frac{\rho}{2} \cdot v^2 \cdot \left[\lambda_R \frac{l}{d} + \sum_{j=1}^N \zeta_j \right]$$

- Varying cross section A_i : Use sectional velocities $v_i = \dot{V}/A_i$!
- Note: In hydraulic systems no superposition law, because non-linear dependence of pressure drop on speed: $\Delta p \sim v^2$.
- Total pressure drop Δp_{res} = need of generated pressure by pump / fan !

$$\Delta p_{res} \sim \frac{\rho}{2} \cdot v^2 \sim \frac{\rho}{2} \cdot \dot{V}^2$$



2. Heating and cooling of electrical machines

Different fluids in hydraulic system (turbulent flow)

- Total pressure drop Δp_{res} : $\Delta p_{res} \sim \frac{\rho}{2} \cdot v^2 \sim \frac{\rho}{2} \cdot \dot{V}^2$
- Generated pressure difference by shaft-mounted fan: $\Delta p_V \sim \frac{\rho}{2} \cdot n^2$
- With $\Delta p_{res} = \Delta p_V$ mass density ρ cancels:

$$v, \dot{V} \neq f(\rho)$$

- The flow rate \dot{V} is depending on speed of machine n , but **nearly not on type of gaseous fluid** (be it air, H_2 etc.)
- Hence it is **also independent of static pressure in gas**: e.g.: pressure of H_2 is 1, 2 or x bar).
- **Flow rate and velocity depend on rotor speed**: $v, \dot{V} \sim n$
e.g.: half speed = half flow rate = half fluid velocity !



2. Heating and cooling of electrical machines

2.7 Windage losses (= ventilation losses)

- Ventilation losses P_{Vent} = sum of

- a) **Power demand for cooling system:** P_F (= e.g. fan input power P_V)
- b) **Surface friction losses** due to gaseous fluid within machine P_{OR} .

$$P_{Vent} = P_F + P_{OR}$$

a) **Power demand for cooling system:** $P_F = \dot{V} \cdot \Delta p / \eta$

- *Different gaseous fluids:* $P_{F,G} \sim \rho_G \sim p$

Due to law of ideal gases mass density rises with static gas pressure !

p / bar	1	2	3	4	5	6
$P_{F,H2}(p)/P_{F,L1}$	0.107	0.214	0.320	0.427	0.534	0.641

Compare power demand for:
a) Air as coolant at $p = 1$ bar: power demand: $P_{F,L1}$
b) Hydrogen gas H₂ (96 % Volume percentage, 50 °C) in dependence of static pressure p



2. Heating and cooling of electrical machines

Surface friction losses

b) Surface friction losses:

Calculation of P_{OR} difficult - experimental help needed !

Example:

Rotors of turbine generators:

Long rotating cylinders, surface $A = d \cdot \pi \cdot l$ (m^2), circumference speed $u = d \cdot \pi \cdot n$ (m/s): Measured losses (kW):

$$P_{OR} = k_{OR} \cdot A \cdot u^3$$

Coefficient k_{OR} : turbulent flow also near surface: $k_{OR} \sim \rho_G \cdot Re^{-0.2}$

Results from experiment at turbine generator rotors, typical air gap widths, static pressure $p = 1$ bar: $k_{OR} = (1.8 \dots 3 \dots 4) \cdot 10^{-6} \text{ kW s}^3 / \text{m}^5$.
(depending on roughness of rotor surface !)



2. Heating and cooling of electrical machines

Coolant gas influence on surface friction losses

- Influence of gas parameters and static pressure on P_{OR} :

- mass density $\rho_G \sim p$,
- kinematic viscosity: $\nu \sim 1/p$

$$P_{OR,G} \sim \rho_G \cdot \nu_G^{0.2} \sim p^{0.8}$$

Compare surface friction losses for:

a) Air as coolant at $p = 1$ bar: $P_{OR,L1}$

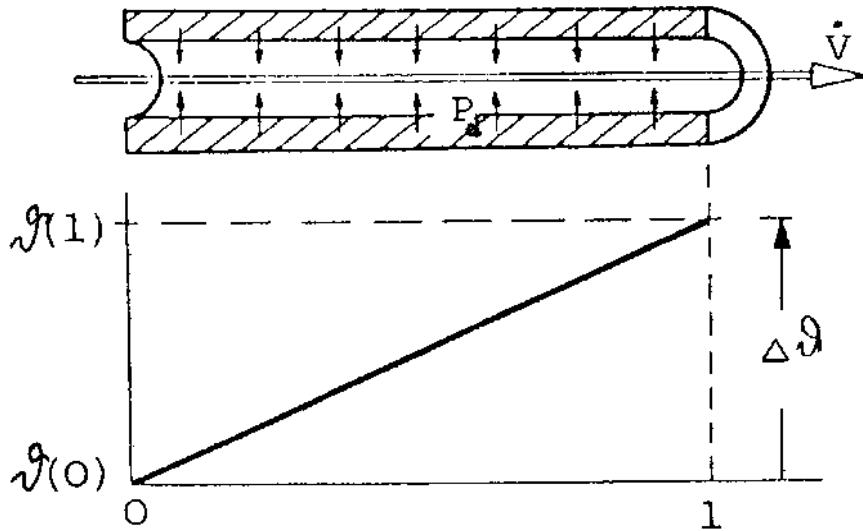
b) Hydrogen gas H_2 (96 % Volume percentage, 50 °C) in dependence of static pressure p

p / bar	1	2	3	4	5	6
$P_{OR,H2}(p)/P_{OR,L1}$	0.148	0.258	0.357	0.449	0.537	0.622



2. Heating and cooling of electrical machines

2.8 Heat transport by coolant



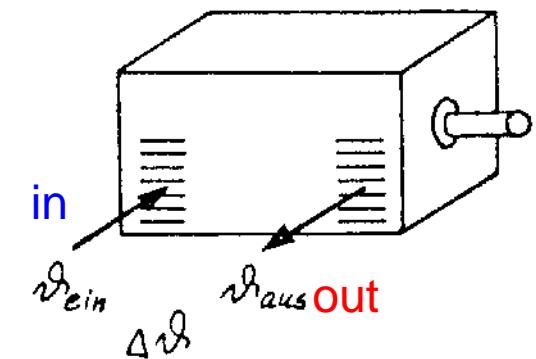
Coolant flow is heated up by losses P_d and increases its temperature:

Temperature rise: $\Delta\vartheta$

Heat (losses P_d) is transferred to outside cooler via convection !

$$\Delta\vartheta = \frac{P_d}{c \cdot \rho \cdot \dot{V}}$$

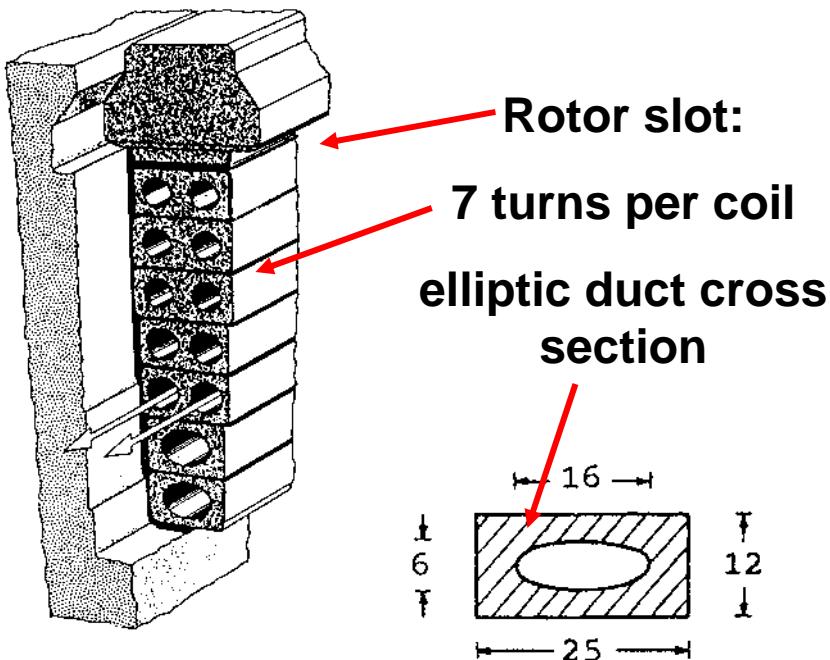
- Typical temperature rise of coolant in machine
(indirect cooled machines): $\Delta\vartheta = 15...35(40)$ K .
- **Typical maximum inlet temperature:** 40 °C.
- Direct cooling: **Maximum outlet temperature:** Gases: 110°C/130°C Thermal Class B/F,
Liquids: H₂O or Oil: 85°C



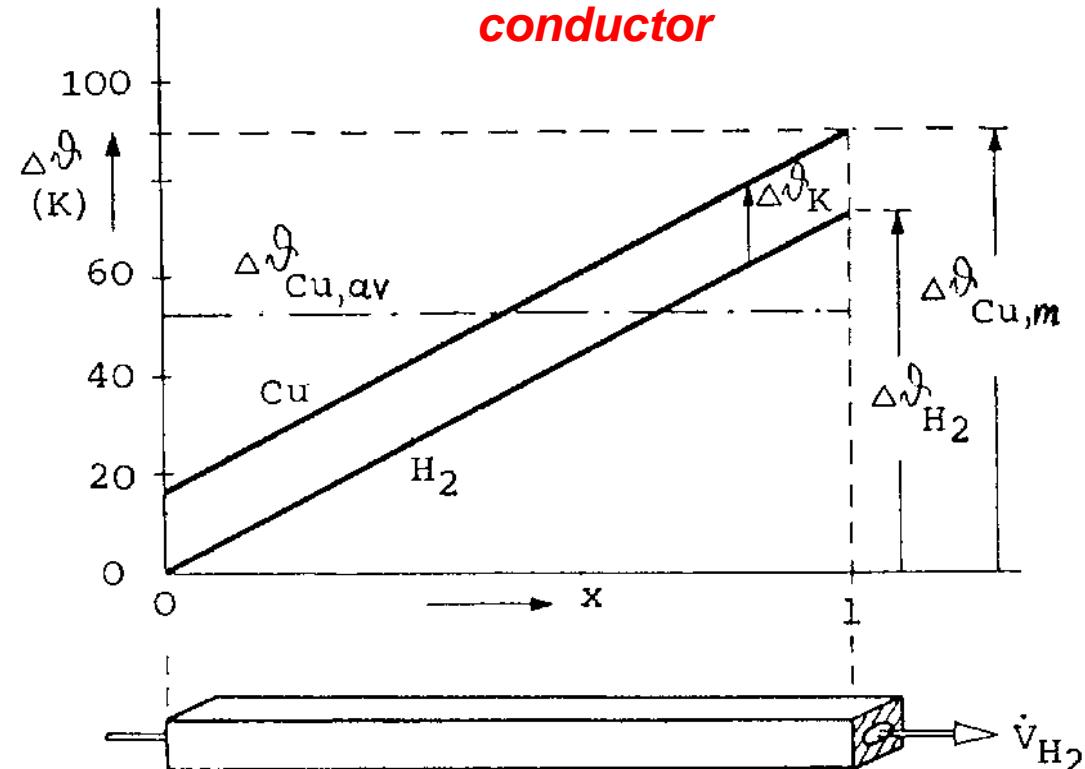
2. Heating and cooling of electrical machines

Hollow rotor conductors of turbine generators directly cooled with hydrogen gas

$$\Delta\vartheta = \frac{P_d}{c \cdot \rho \cdot \dot{V}}$$

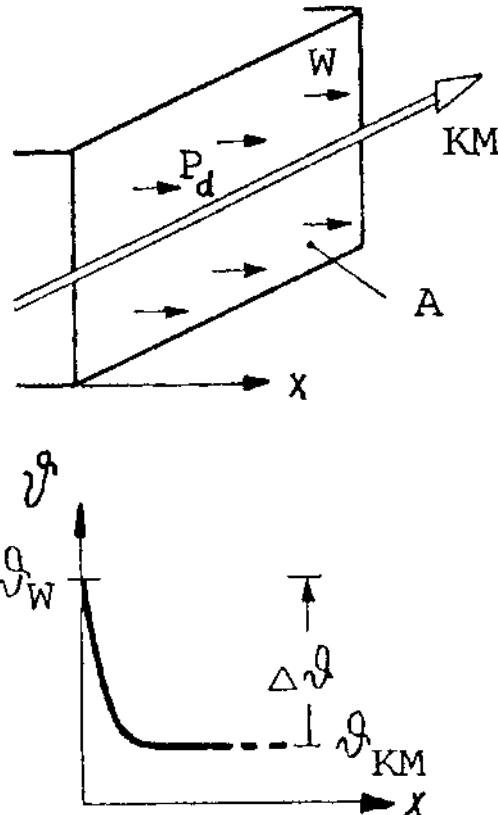


Linear temperature rise along conductor



2. Heating and cooling of electrical machines

2.9 Heat transfer



a) Radiation

Small effect, as $\Delta\vartheta$ usually < 100 K between machine surface and ambient: $\alpha_s \approx 7 \text{ W}/(\text{m}^2\text{K})$.

b) Convection:

Coolant is heated up, transports off the heat by its movement !

Hot surface: temperature ϑ_w , cool coolant: temperature ϑ_{KM}

Surface: A , heat flow: P_d !

$$P_d = \alpha_k A (\vartheta_w - \vartheta_{KM})$$

Heat transfer coefficient of convection: α_k

Free (natural) convection: Lower mass density of hot coolant gives rise to coolant circulation : $\alpha_k \approx 7 \text{ W}/(\text{m}^2\text{K})$

In this case radiation has to be considered: $\alpha_k + \alpha_s = 15 \text{ W}/(\text{m}^2 \text{ K})$

2. Heating and cooling of electrical machines

Forced convective heat transfer

Forced convection:

Coolant is moved by pump or fan with rather high velocity, so that flow is turbulent. This yields **high heat transfer coefficients α_K !**

Turbulent flow gives higher α_K than laminar flow !

a) **Turbulent LIQUID flow in tubes** (e.g. water in hollow conductors):
Typical ratio length/diameter of tube: $l/d = 100 \dots 400$ and $Re > 10^4$:

$$\alpha_K \approx 0.024 \cdot (c \cdot \rho)^{0.3} \cdot \frac{\lambda^{0.7}}{d^{0.2} v^{0.5}} \cdot v^{0.8}$$

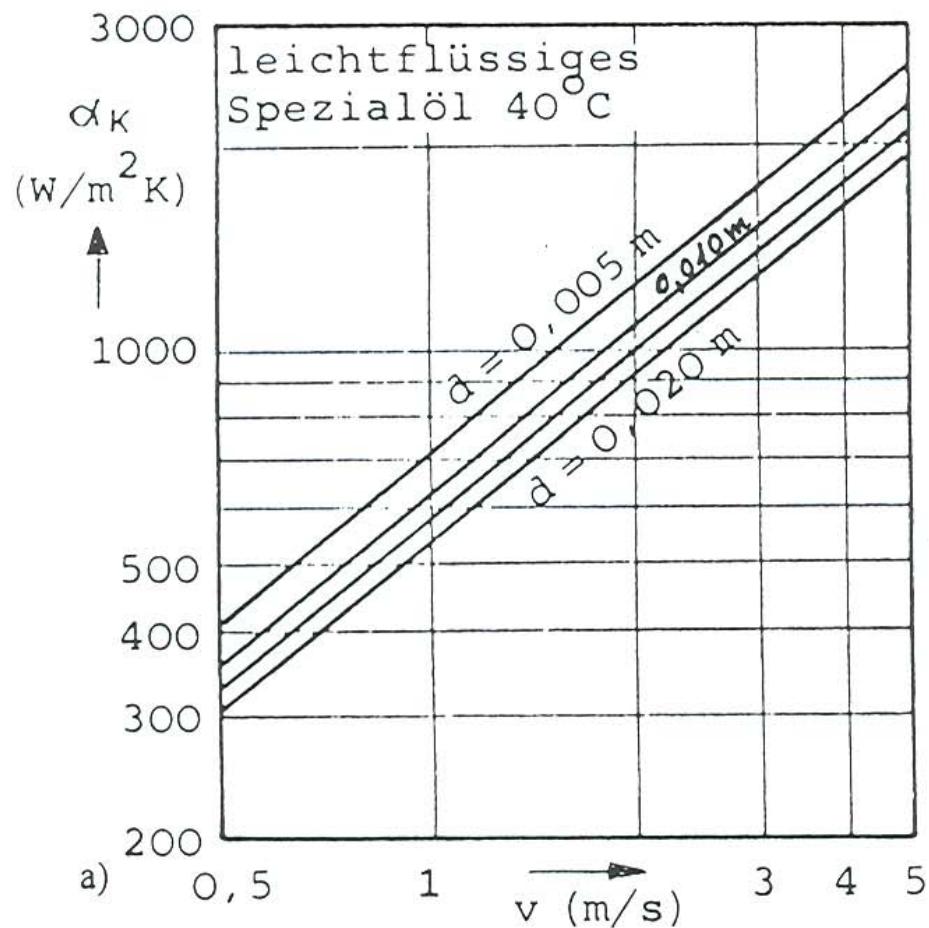
b) **Turbulent gas flow in tubes:** (above $Re > 10^4$)

$$\alpha_K \approx 0.027 \cdot (c \cdot \rho)^{0.78} \cdot \left(\frac{\lambda}{d} \right)^{0.22} \cdot v^{0.78}$$

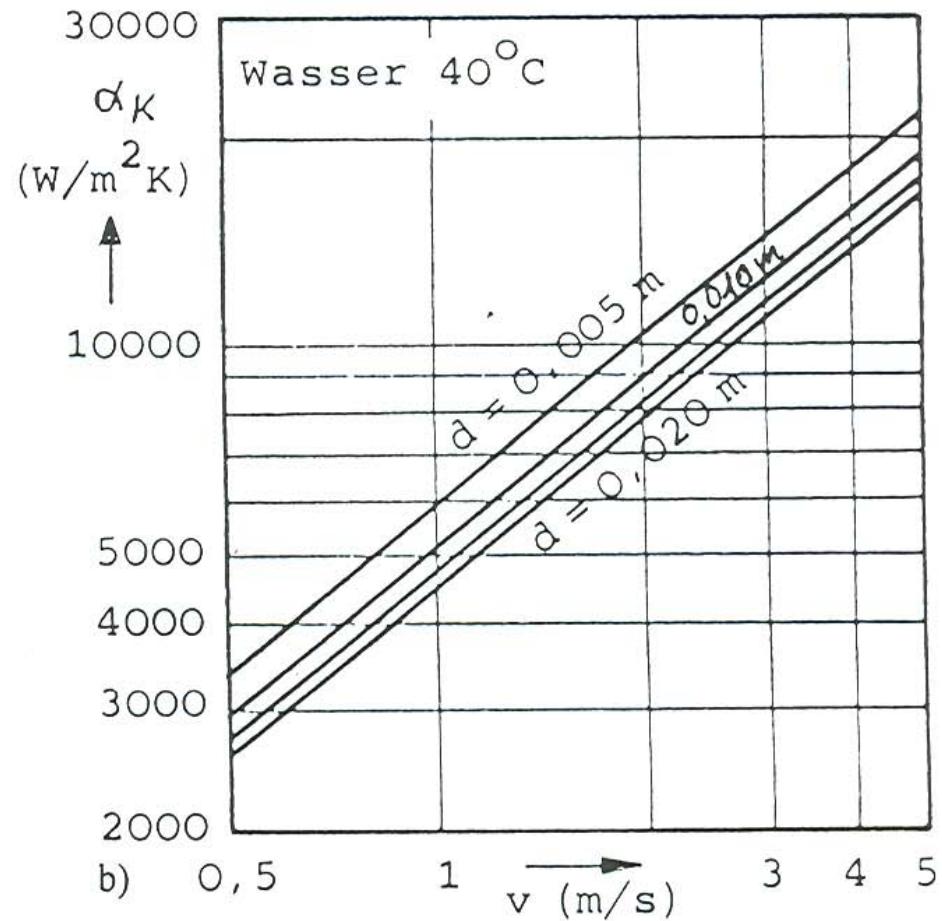


2. Heating and cooling of electrical machines

Heat transfer coefficient for turbulent LIQUID flow in tubes at 40°C



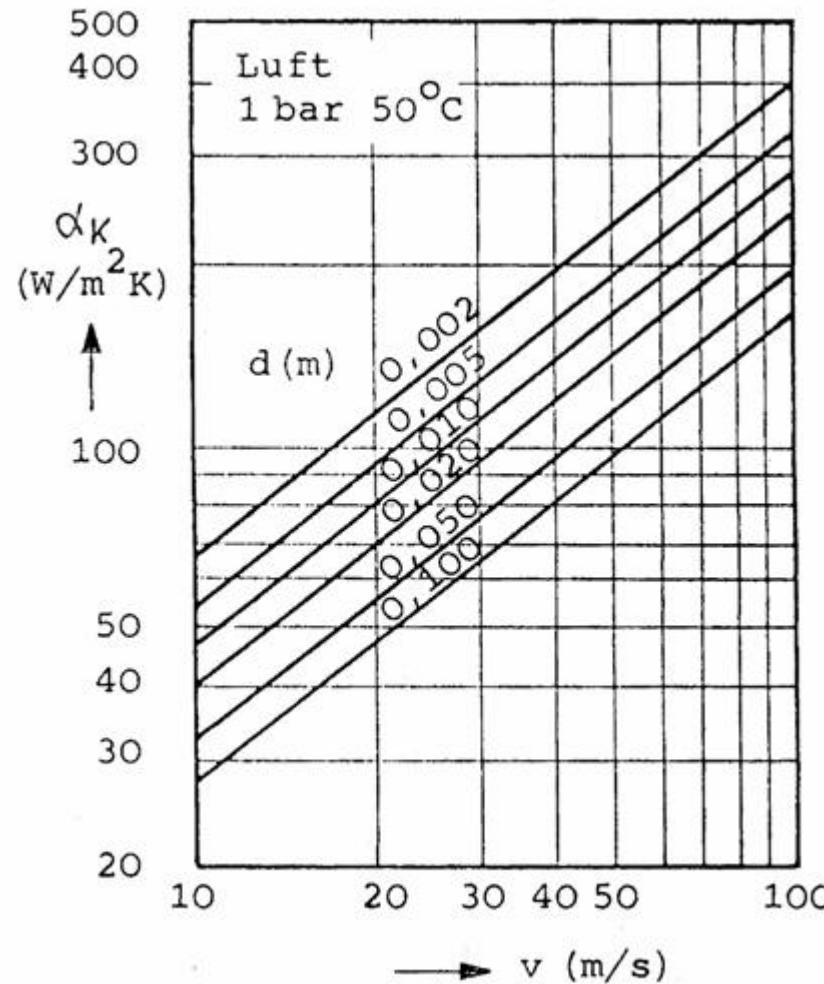
a) special low-viscous oil



b) water

2. Heating and cooling of electrical machines

Heat transfer coefficient for turbulent AIR flow in tubes at 50°C



Static
pressure:
1 bar

2. Heating and cooling of electrical machines

Heat transfer coefficient for different gases in tubes at 50°C

For other gases than air heat transfer coefficient changes due to

$$\alpha_K \approx 0.027 \cdot (c \cdot \rho)^{0.78} \cdot \left(\frac{\lambda}{d} \right)^{0.22} \cdot v^{0.78}$$

and $\rho \sim p$ with:

$$\alpha_K \sim (c \cdot \rho)^{0.78} \cdot \lambda^{0.22} \sim p^{0.78}$$

Compare heat transfer coefficient (HTC) for:

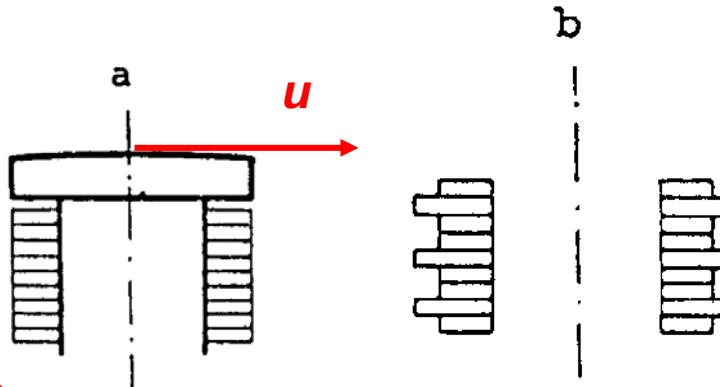
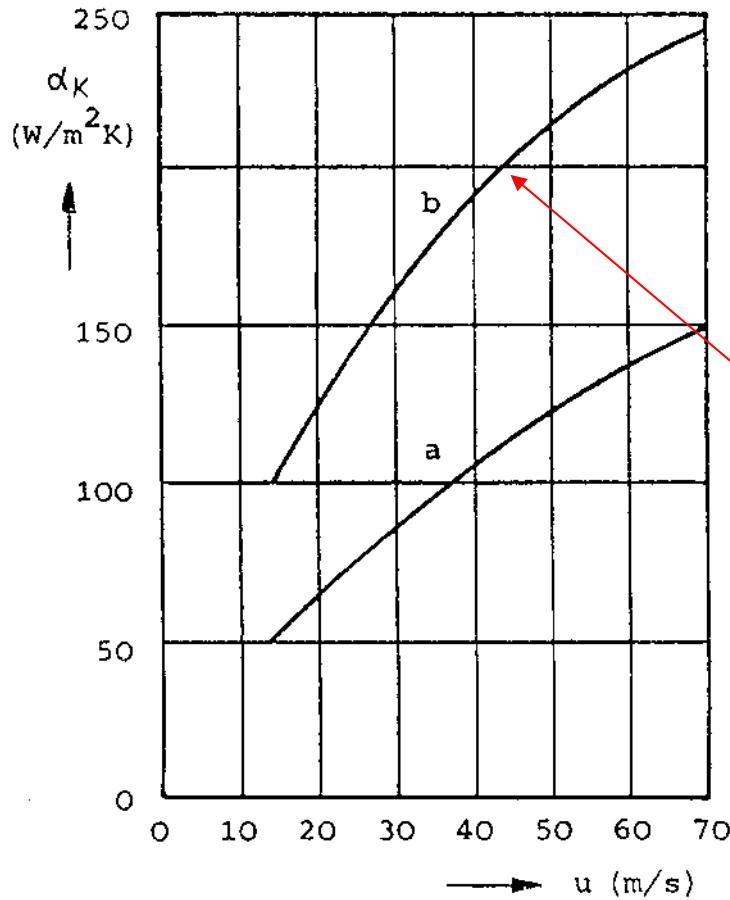
- Air as coolant at $p = 1$ bar: HTC: $\alpha_{K,L1}$
- Hydrogen gas H₂ (96 % Volume percentage, 50 °C) in dependence of static pressure p

p / bar	1	2	3	4	5	6
$\alpha_{KH2}(p)/\alpha_{KL1}$	1.49	2.56	3.51	4.40	5.23	6.03



2. Heating and cooling of electrical machines

Heat transfer coefficient for salient pole synchronous machines in turbulent air flow



Flat copper coils Flat copper coils with cooling fins

In **case b)** surface A is increased! Taking surface A of **case a)**, therefore bigger HTC has to be used !

Rotor circumference speed $u = d_r \cdot \pi \cdot n$:
HTC is in **case b)** virtually increased due to
bigger cooling surface A !

2. Heating and cooling of electrical machines

“Cooling fins” by broader copper turns in salient pole excitation winding



2. Heating and cooling of electrical machines

Summary: Heat transfer coefficients (W/(m²K))

Natural convection	Forced convection
15	Air 50 ... 300
	Hydrogen gas 100 ... 1500
	Oil 500 ... 2000
	Water 5000 ... 20000



2. Heating and cooling of electrical machines

Comparison of hydrogen gas and air as coolant

- a) Air as coolant at $p = 1$ bar
- b) Hydrogen gas H_2 (96 % Volume percentage, 50 °C) in dependence of static pressure p

a) Power demand for cooling system:

p / bar	1	2	3	4	5	6
$P_{F,H2}(p)/P_{F,L1}$	0.107	0.214	0.320	0.427	0.534	0.641

b) Surface friction losses:

$P_{OR,H2}(p)/P_{OR,L1}$	0.148	0.258	0.357	0.449	0.537	0.622
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c) Heat transfer coefficient:

$\alpha_{KH2}(p)/\alpha_{KL1}$	1.49	2.56	3.51	4.40	5.23	6.03
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d) Specific heat capacity:

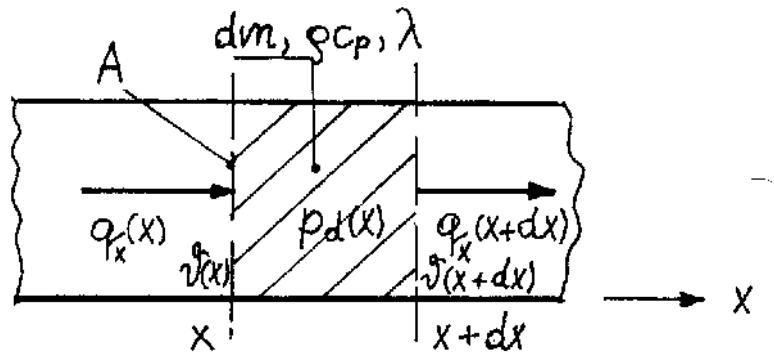
$c\rho_{H2}(p)/c\rho_{L1}$	1	2	3	4	5	6
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2. Heating and cooling of electrical machines

2.10 Conduction of heat

- Fourier's law of conduction of heat: $q = -\lambda \cdot \text{grad } \vartheta = -\lambda \cdot (\partial \vartheta / \partial x, \partial \vartheta / \partial y, \partial \vartheta / \partial z)$
- Heating up of mass element dm : $dm \cdot c \cdot (d\vartheta(x) / dt) = p_d(x) \cdot dV + q_x(x)A - q_x(x+dx)A$



$$\rho \cdot c \cdot (d\vartheta(x) / dt) = p_d(x) - \partial q_x(x) / \partial x$$

- Combining both laws:

$$\rho \cdot c \cdot (d\vartheta(x) / dt) = p_d(x) + \lambda \cdot \partial^2 \vartheta(x) / \partial x^2$$

- For three co-ordinates x, y, z :

$$\rho \cdot c \cdot \frac{\partial \vartheta}{\partial t} = p_d(x, y, z) + \lambda_x \cdot \frac{\partial^2 \vartheta}{\partial x^2} + \lambda_y \cdot \frac{\partial^2 \vartheta}{\partial y^2} + \lambda_z \cdot \frac{\partial^2 \vartheta}{\partial z^2}$$

- Partial differential equation of „heat transportation“ in thermally conductive media with anisotropy: Solution: $\vartheta(x, y, z, t)$: „Time-varying“ temperature field !

2. Heating and cooling of electrical machines

Special cases of conduction of heat

a) **Stationary temperature field** (no dependence on time: $d./dt = 0$):
(POISSON equation).

$$\lambda_x \cdot \frac{\partial^2 \vartheta}{\partial x^2} + \lambda_y \cdot \frac{\partial^2 \vartheta}{\partial y^2} + \lambda_z \cdot \frac{\partial^2 \vartheta}{\partial z^2} = -p_d(x, y, z)$$

b) **Isotropy of material** ($\lambda = \lambda_x = \lambda_y = \lambda_z$), **no heat sources**: ($p_d = 0$) :
LAPLACE equation:

$$\frac{\partial^2 \vartheta}{\partial x^2} + \frac{\partial^2 \vartheta}{\partial y^2} + \frac{\partial^2 \vartheta}{\partial z^2} = 0$$

- Partial differential equation of „heat transportation“ in thermally conductive media with isotropy: Solution: $\vartheta(x, y, z)$: „*Time-invariant*“ temperature field !



2. Heating and cooling of electrical machines

<u>Metals</u>	λ (W/mK)	<u>Insulating materials</u>	λ (W/mK)
Cast iron	30... 46	Glass	0.8 ... 1.2
Steel	40... 46	Asbestos	ca. 0.2
Nirosta Steel	25... 30	Mica	0.4 ... 0.6
Non-magnetic steel	14... 16	Paper	0.05...0.15
Si-alloy steel sheets	15... 48	Polyamide-paper (Nomex) ca. 0.13	
Pure "electrolytic" copper	ca. 390	Pressed wood	0.08...0.2
"Technical" copper	ca. 380	Wood	0.14...0.3
Brass	ca. 110	Hard board	0.23...0.28
Bronze	ca. 100	Mica foil HV insulation	0.15...0.2
Zinc	ca. 110	Mica-Resin compound	0.2 ... 0.3
Lead	ca. 35	Epoxy resin	0.17...0.23
Pure aluminium	ca. 220	Resin impregnated	
Aluminium alloys	100...190	press board	0.2 ... 0.5
		Teflon	0.2 ... 0.24

At 50° C.

At 100° C : By ca. 1...2 % lower

At 50° C.

At 100° C: Increase by ca. 10%.



2. Heating and cooling of electrical machines

Law of Wiedemann-Franz-Lorenz

- Pure metals: Temperature is proportional to kinetic energy of “free” electrons = Conduction of heat (**Fourier**’s law) is done by moving free electrons. $q = -\lambda \cdot \text{grad } \vartheta$

As current flow (= conduction of electric charges (**Ohm**’s law)) is also done by “free” electrons, both laws are similar:

$$J = \kappa \cdot E = -\kappa \cdot \text{grad } \varphi$$

Electric and thermal conductivity are proportional: $\lambda \sim \kappa$

	Copper	Pure aluminium	Pure iron
$\kappa / \text{S/m (at } 20^\circ\text{C)}$	$57 \cdot 10^6$ (100%)	$34 \cdot 10^6$ (60%)	$10 \cdot 10^6$ (18%)
$\lambda / \text{W/(m}\cdot\text{K) (50}^\circ\text{C)}$	380 (100%)	220 (58%)	48 (13%)

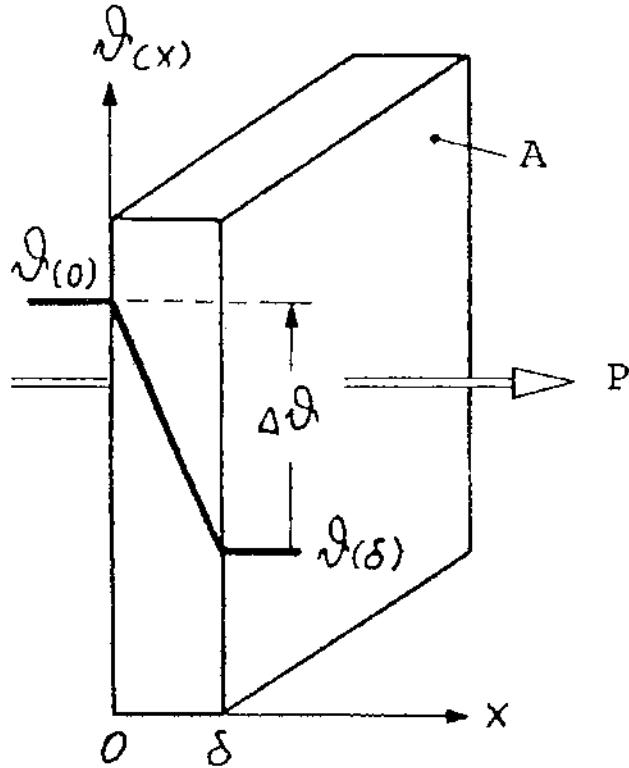


2. Heating and cooling of electrical machines

One-dimensional heat conduction in a plate

Plate: Thickness δ , area A , heat flow $q \cdot A = P$

$$q_x = -\lambda \cdot (\partial \vartheta / \partial x)$$



$$q = \frac{P}{A} = -\lambda \frac{d\vartheta}{dx} \rightarrow \frac{d\vartheta}{dx} = -\frac{P}{\lambda A}$$

Boundary condition: at $x = 0$ is $\vartheta = \vartheta(0)$:

Result:

$$\vartheta(x) = \vartheta(0) - \frac{P}{\lambda A} x$$

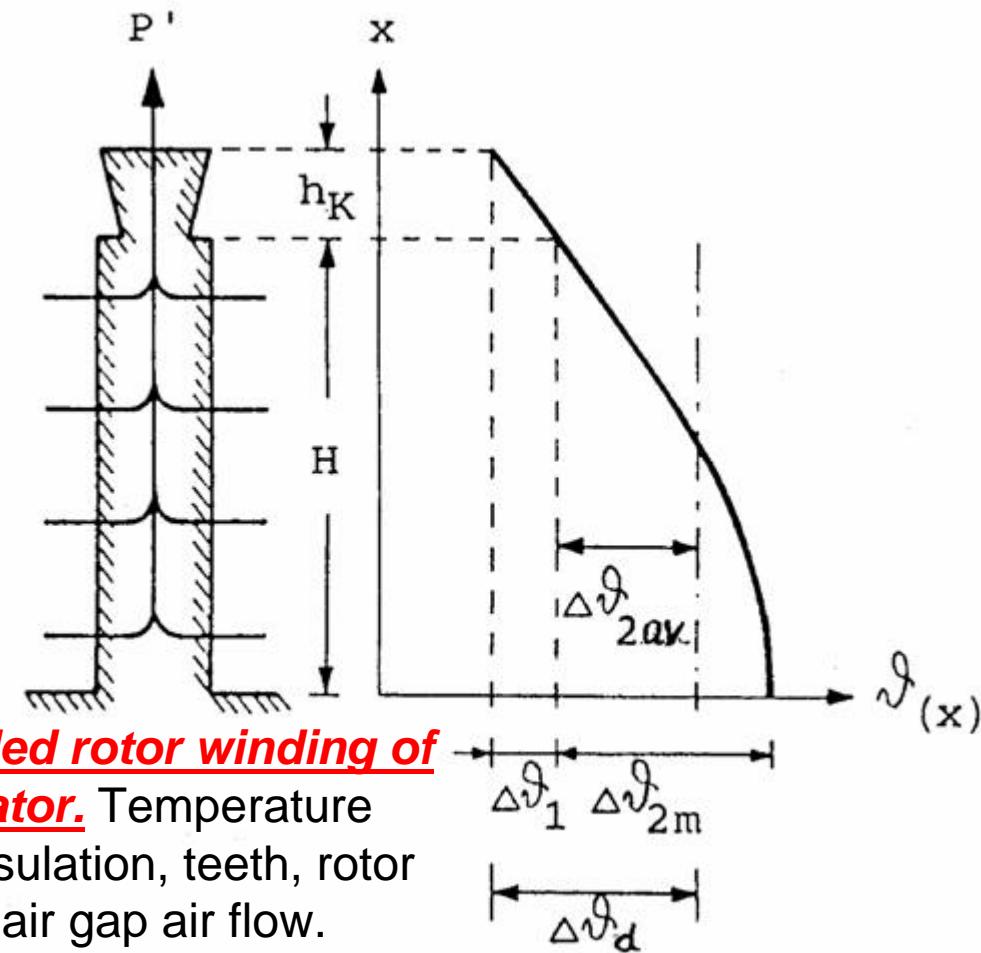
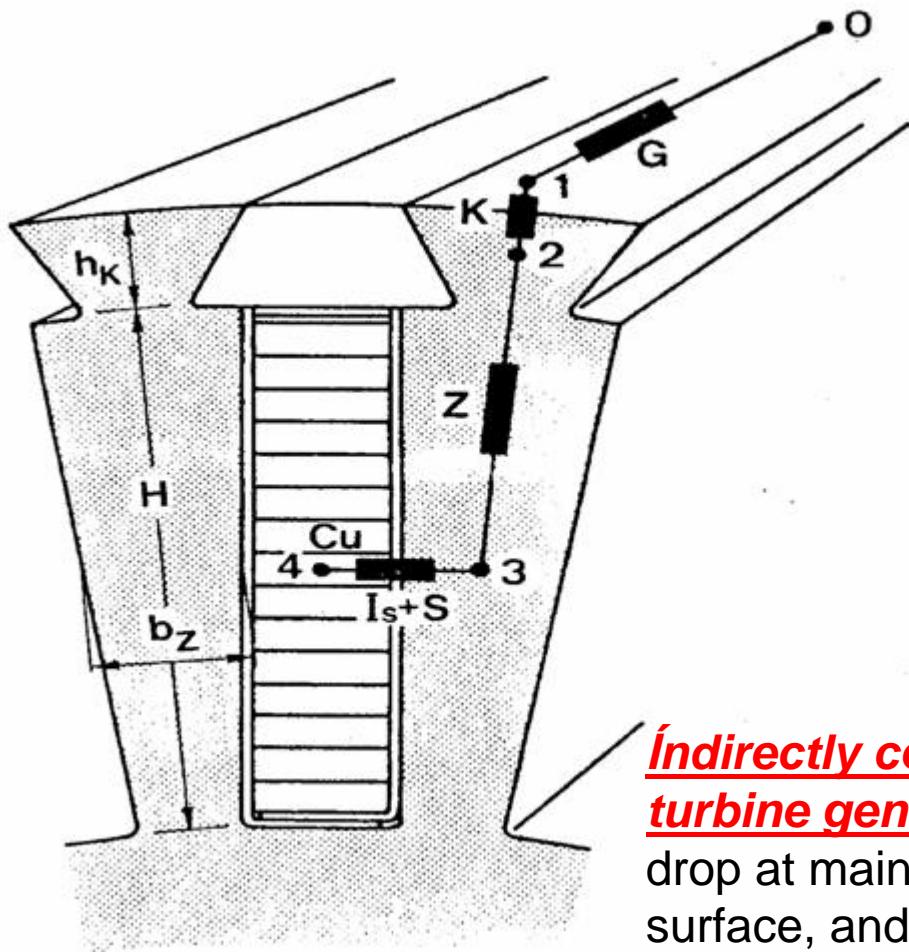
- Linear drop of temperature in the plate !

- OHM's law of heat conduction: $\Delta \vartheta = R \cdot P$

- Thermal resistance: $R = \frac{\delta}{\lambda A}$

2. Heating and cooling of electrical machines

Temperature drop in slot main insulation



Indirectly cooled rotor winding of turbine generator. Temperature drop at main insulation, teeth, rotor surface, and in air gap air flow.

2. Heating and cooling of electrical machines

Heat conduction in laminated structure

- **Example:** Two plates of different material = laminated structure
- **Two thermal resistances** R_1 und R_2 ,
- **identical area** A = series connection of thermal resistances

$$R_{res} = \frac{1}{A} \cdot \left(\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} \right)$$

- Equivalent structure of total thickness $\delta = \delta_1 + \delta_2$:

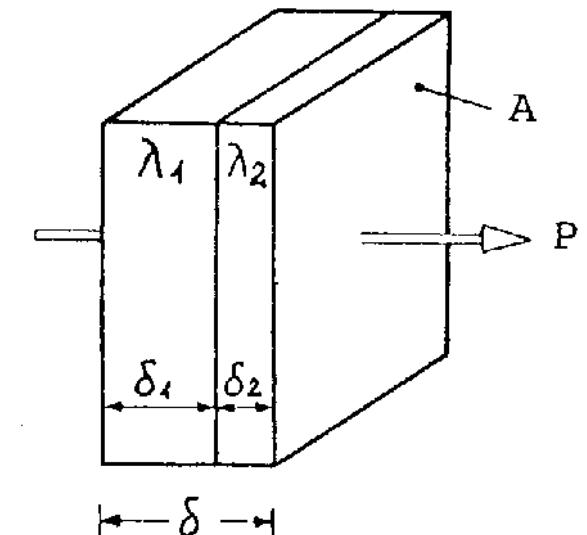
$$R_{res} = \frac{1}{A} \cdot \frac{\delta_1 + \delta_2}{\lambda_{res}}$$

- **Equivalent thermal conductivity** λ_{res} :

$$\lambda_{res} = \frac{\delta_1 + \delta_2}{\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2}}$$

with N layers:

$$\lambda_{res} = \frac{\sum_{i=1}^N \delta_i}{\sum_{i=1}^N \frac{\delta_i}{\lambda_i}}$$



2. Heating and cooling of electrical machines

Heat conduction in iron stack

- Perpendicular to sheet plane of iron stack we have a multi-layer laminated structure of iron sheets, sheet insulation and enclosed air.
- Transverse thermal conductivity λ_q depends of pressure of stack, which eliminates air.

Example:

Iron sheet with $v_{10} = 1.7 \text{ W/kg}$ (1 T, 50 Hz)

Iron:	$\delta_{Fe} = 0.5 \text{ mm},$	$\lambda_{Fe} = 18 \text{ W/(m}\cdot\text{K)}$
Enamel insulation:	$\delta_{Is} = 0.02 \text{ mm},$	$\lambda_{Is} = 0.3 \text{ W/(m}\cdot\text{K)}$
Enclosed air layer:	$\delta_{Air} = 0.0015 \text{ mm},$	$\lambda_{Air} = 0.027 \text{ W/(m}\cdot\text{K)}$

Resulting transverse thermal conductivity: $\lambda_q = 3.4 \text{ W/(m}\cdot\text{K)}$

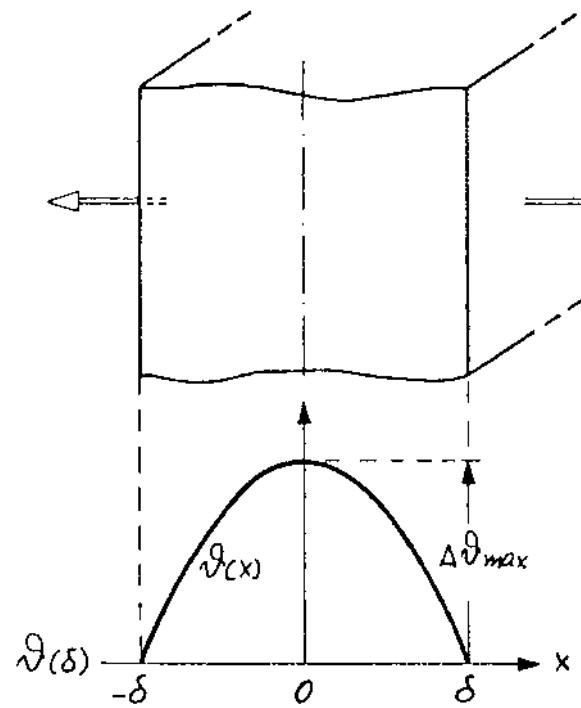
Fact:

Transverse to sheet plane thermal conductivity is smaller by a factor of 5 than in sheet plane.



2. Heating and cooling of electrical machines

Axial temperature distribution in iron stack (1)



Iron losses are distributed heat source in iron stack p_d !

1st integration:
$$\frac{d^2\vartheta}{dx^2} = -\frac{p}{\lambda} \rightarrow \frac{d\vartheta}{dx} = -\frac{p}{\lambda} \cdot x + C_0$$

1. boundary condition:

No heat flow at symmetry line $x = 0$:

$$q(0) = -\lambda \cdot \frac{d\vartheta}{dx} \Big|_{x=0} = 0 \rightarrow C_0 = 0$$

2nd integration:

$$\vartheta(x) = -\frac{p}{\lambda} \cdot \frac{x^2}{2} + C_1$$

2. boundary condition: $\vartheta(-\delta) = \vartheta(\delta)$

$$C_1 = \vartheta(\delta) + \frac{p}{\lambda} \cdot \frac{\delta^2}{2}$$



2. Heating and cooling of electrical machines

Axial temperature distribution in iron stack

Solution: Parabolic axial temperature distribution:

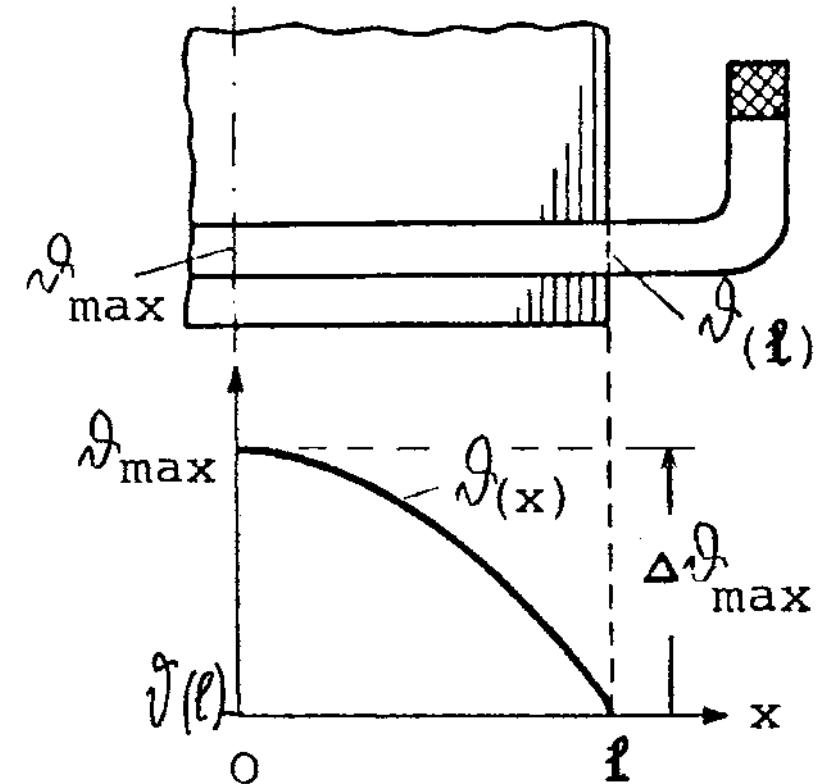
Maximum in middle of stack at $x = 0$:

$$\vartheta(x) = \vartheta(\delta) + \frac{P}{2\lambda} \cdot (\delta^2 - x^2)$$

At large stack lengths temperature rise in the **middle is high**

- due to parabolic temperature distribution
- due to low $\lambda_q = \lambda_r/5$!

Necessity to segment the stack with radial ventilation ducts !



2. Heating and cooling of electrical machines

2.11 Efficiency of cooling systems

a) „Specific heat capacity“: $P = c \cdot \rho \cdot A \cdot v \cdot \Delta \vartheta$ "c · ρ · v"

v: gases 50 m/s, H₂O in Cu-hollow conductors 1.5 m/s

b) Convection: HTC: α_K $P = \alpha_K \cdot O \cdot \Delta \vartheta_K$

v see a): hydraulic diameter $d = 0.01$ m

c) Power demand: $P_F = \dot{V} \cdot \Delta p_{res} / \eta = \frac{P_d}{c \cdot \rho \cdot \Delta \vartheta} \cdot \frac{\rho}{2} \cdot v^2 \cdot \zeta_{res} \cdot \frac{1}{\eta} \sim \frac{1}{c\rho} \cdot \rho \cdot v^2 \cdot \zeta_{res}$

	$c\rho / \text{Ws}/(\text{m}^3\text{K})$	$\rho / \text{kg/m}^3$	$v / \text{m/s}$	ζ_{res}
Water, 60°C	$4123 \cdot 10^3$	983	1.5	150
Air, 50°C	1065	1.058	30	10

$$P_{F,H2O}/P_{F,L} = 0.009 = 0.9\%$$



2. Heating and cooling of electrical machines

Coolant	Material data		Cooling effect		Power demand	
Per unit of effect of AIR	Mass density	Specific heat	Specific heat transfer	Heat transfer coefficient	Fan power	Surface friction losses
	ρ / ρ_L	$c\rho / (c\rho)_L$	$c\rho v / (c\rho v)_L$	$\alpha_K / \alpha_{K,L}$	$P_F / P_{F,L}$	$P_{OR} / P_{OR,L}$
Air, 1 bar	1.0	1.0	1.0	1.0	1.0	1.0
H ₂ , 96% 1 bar	0.107	≈ 1.0	≈ 1.0	1.49	0.107	0.148
2 bar	0.214	≈ 2.0	≈ 2.0	2.56	0.214	0.258
4 bar	0.427	≈ 4.0	≈ 4.0	4.40	0.427	0.449
He, 1 bar	0.138	0.74	0.74	1.17	0.138	0.209
CO ₂ , 1 bar	1.528	1.27	1.27	1.08	1.528	1.344
N ₂ , 1 bar	0.967	1.03	1.03	1.02	0.967	0.966
Water	935	3880	116	43	0.1...1%	Given by air gap gas
Low viscous oil	740	1550	47	5	0.1...1%	Given by air gap gas



2. Heating and cooling of electrical machines

2.12 Transient heat flow

- Heat balance for mass M during time interval dt :

$$P \cdot dt = (\vartheta - \vartheta_0) \cdot dt / R + M \cdot c \cdot d\vartheta$$

Losses heat transfer to ambient stored heat in body

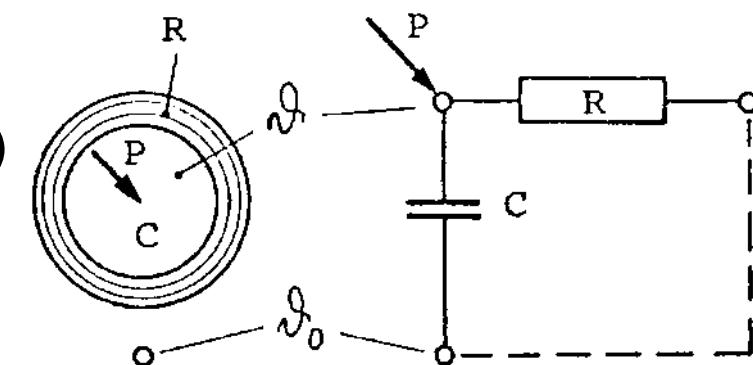
- Differential equation:

with concentrated elements: $C = M \cdot c$, $R = 1 / (\alpha_K A)$

$$\vartheta + C \cdot R \cdot d\vartheta / dt = \vartheta_0 + P \cdot R$$

- Corresponding partial differential equation for distributed material properties:

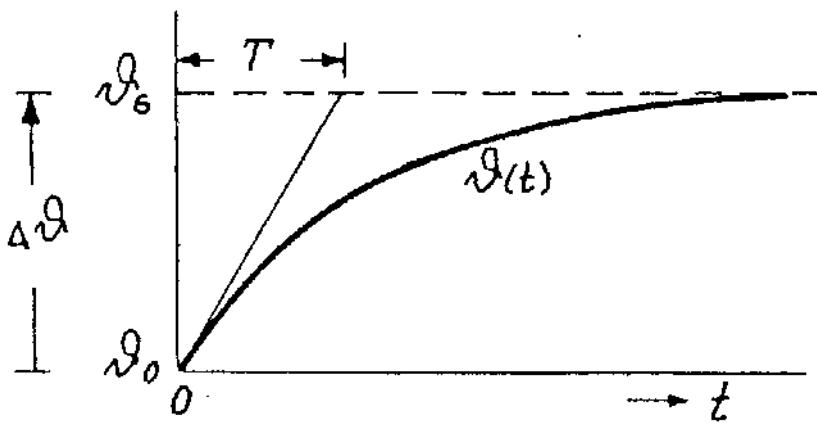
$$\rho \cdot c \cdot \frac{\partial \vartheta}{\partial t} = p_d(x, y, z) + \lambda_x \cdot \frac{\partial^2 \vartheta}{\partial x^2} + \lambda_y \cdot \frac{\partial^2 \vartheta}{\partial y^2} + \lambda_z \cdot \frac{\partial^2 \vartheta}{\partial z^2}$$



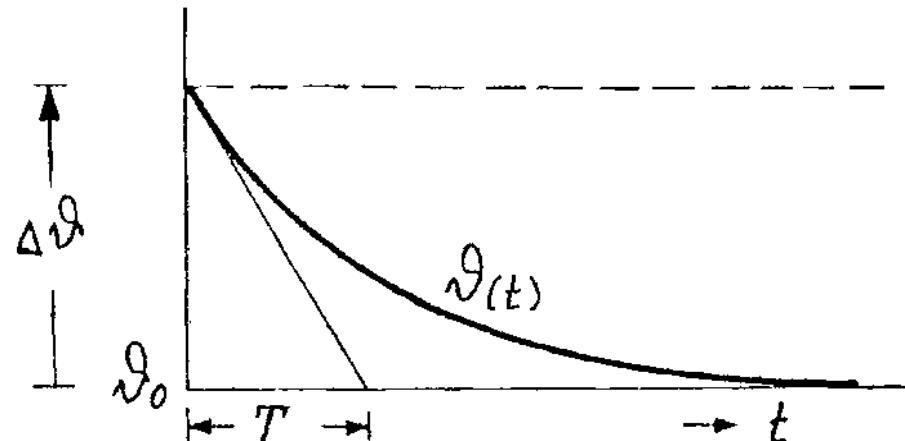
„homogeneous-body“ replica

2. Heating and cooling of electrical machines

Heating up & cooling down



$$\theta(t) = \theta_0 + (\theta_s - \theta_0) \cdot (1 - e^{-t/T})$$



$$\theta(t) = \Delta\theta \cdot e^{-t/T} + \theta_0$$

- Thermal time constant of „homogeneous-body“ replica $T = C \cdot R = M \cdot c / (\alpha_K \cdot A)$
- Steady state temperature rise: $\Delta\theta_s = P \cdot R = P / (\alpha_K \cdot A)$
- Adiabatic heating = no heat exchange
= tangent to temperature rise curve:

$$\left. \frac{d\theta}{dt} \right|_{t=0} = \frac{\theta_s - \theta_0}{T} = \frac{P}{Mc}$$

2. Heating and cooling of electrical machines

Adiabatic heating

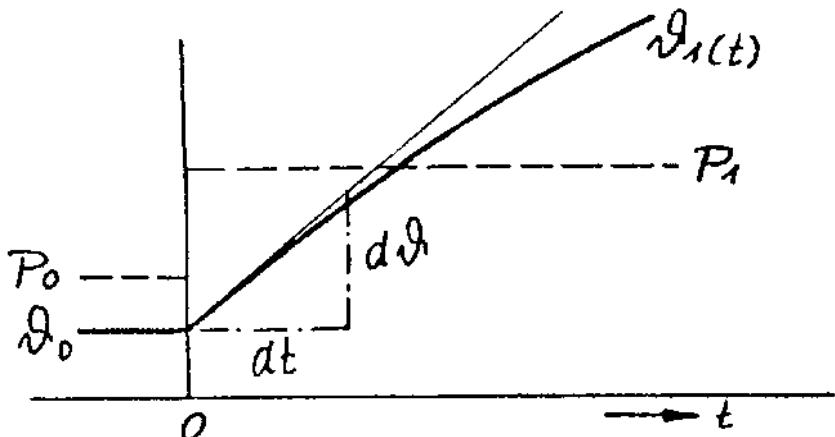
- Adiabatic heating = no heat exchange = tangent to temperature rise curve:
- Differential equation: $C = M \cdot c$, R is infinite = T is infinite

$$C \cdot d(\vartheta - \vartheta_0) / dt = P$$

Solution: $\vartheta - \vartheta_0 = P/C \cdot t$

- Application: Short time overload:

Duration t much shorter than T , so T is regarded as infinite.



Example:

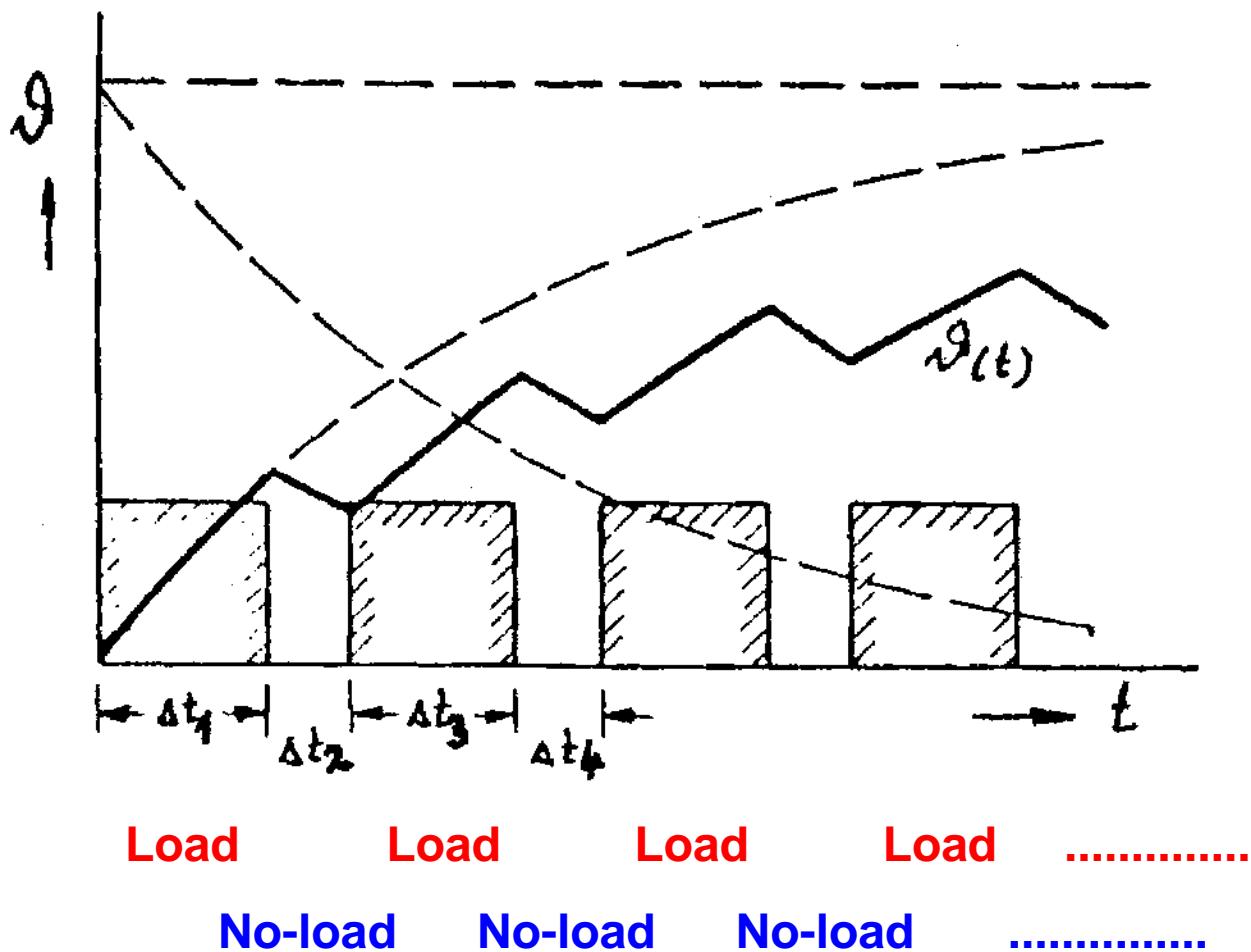
Conductor: Volume V : Rated current density: J_0 , at overload: J_1

$$\text{Additional losses in conductor at overload: } P_1 - P_0 = (J_1^2 - J_0^2) \cdot \frac{V}{\kappa}$$

$$\text{Temperature rise due to heat capacity: } C = c\rho V: \left. \frac{d\vartheta}{dt} \right|_{t=0} = \frac{J_1^2 - J_0^2}{c\rho\kappa}$$

2. Heating and cooling of electrical machines

Intermittent periodic duty S3



2. Heating and cooling of electrical machines

Thermal stability of high-loaded electric conductor

- Increase of specific electric resistance $\rho_{el} = 1/\kappa$ with temperature with coefficient α_g , leads to increased losses, which cause further temperature rise. **Is there a stable solution ?**

$$\rho_{el}(g) = \rho_{el}(g_0) \cdot [1 + \alpha_g(g - g_0)]$$

- Increase of losses:

$$P(g) = J^2 V \rho_{el}(g) = P_0 (1 - \alpha_g g_0) + P_0 \alpha_g \cdot g$$

- Differential equation of “homogeneous body” replica:

$$g \cdot (1 - R \cdot P_0 \alpha_g) + C \cdot R \cdot \frac{dg}{dt} = g_0 + P_0 \cdot (1 - \alpha_g g_0) \cdot R$$

- Time constant depends on losses: $T = \frac{C \cdot R}{1 - R \cdot P_0 \cdot \alpha_g}$

Condition for stability: Time constant must be positive:

$$P(g_0) \cdot R \leq \frac{1}{\alpha_g}$$



2. Heating and cooling of electrical machines

Thermal instability

- In case of **too big losses**:

$$P(\vartheta_0) \cdot R > \frac{1}{\alpha_g}$$

Time constant gets negative $T < 0$, so solution yields exponential increase of temperature rise !

- Solution for $T < 0$: $\vartheta(t) = \vartheta_0 + (\vartheta_s - \vartheta_0) \cdot (1 - e^{-t/|T|}) \sim e^{t/abs(T)}$

Example:

Cu-conductor: ($\alpha_g = 1/255 \text{ 1/K}$, at $\vartheta_0 = 20^\circ\text{C}$)

Overload losses P at 20° C must be below a value, which would lead to stationary temperature rise of $1/\alpha_g = 255 \text{ K}$!

Otherwise: **Fuse effect:** Copper conductor over-heats and evaporates !

