

# 3. Eddy current losses in winding systems

- Additional losses in electrical machines
- Basics on current displacement
- Current displacement in massive slot conductor
- Critical conductor height
- Use of current displacement in electrical machines
- Methods to reduce current displacement effect
- Air-gap winding for superconducting machines



# 3. Eddy current losses in winding systems

## 3.1 Additional losses in electrical machines

### Losses in synchronous machines at sinusoidal line voltage operation

#### A) No-load losses: ( $I_s = 0$ ):

- A1) **Iron losses** (Eddy current & hysteresis losses) in stator iron teeth and yoke
- A2) **Friction losses** (Bearings, brushes)
- A3) **Ventilation (windage) losses** (Fan power, rotor surface friction)
- A4) **Additional no-load losses**

#### B) Losses at load: (occur in addition to A), if $I_s > 0$ ):

- B1) **Ohmic losses in stator winding**
- B2) **Stray load losses (= additional losses at load)**

#### C) Excitation losses:

- C1) **Ohmic losses in rotor field winding**
- C2) **Electric losses in brushes**
- C3) **Losses in exciter (= exciting machine or power electronics)**



# 3. Eddy current losses in winding systems

## Additional no-load losses

- Losses in press plates and press fingers:

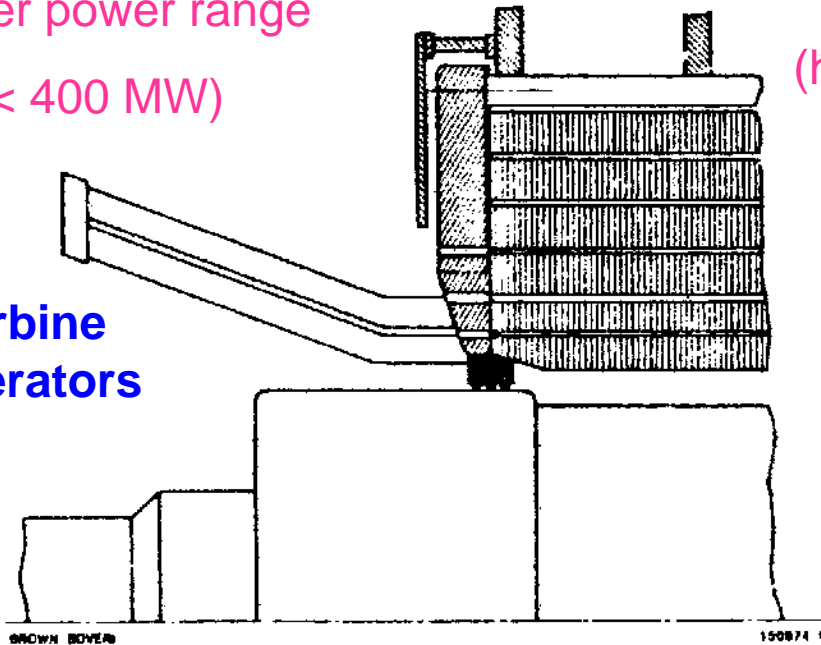
Axial end field component induces with stator frequency eddy currents in press plates

Aluminium press plate

(lower power range

< 400 MW)

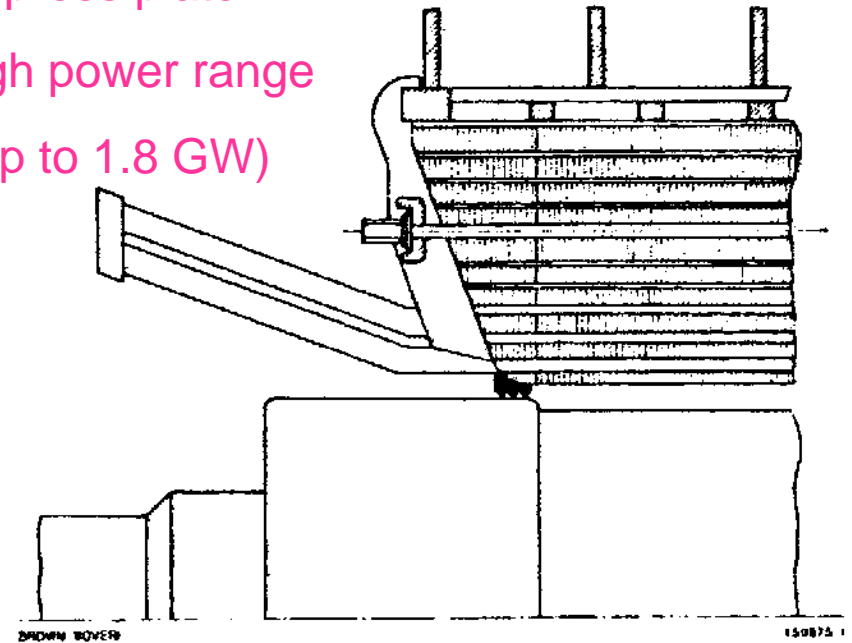
Turbine  
generators



Laminated glued steel  
press plate

(high power range

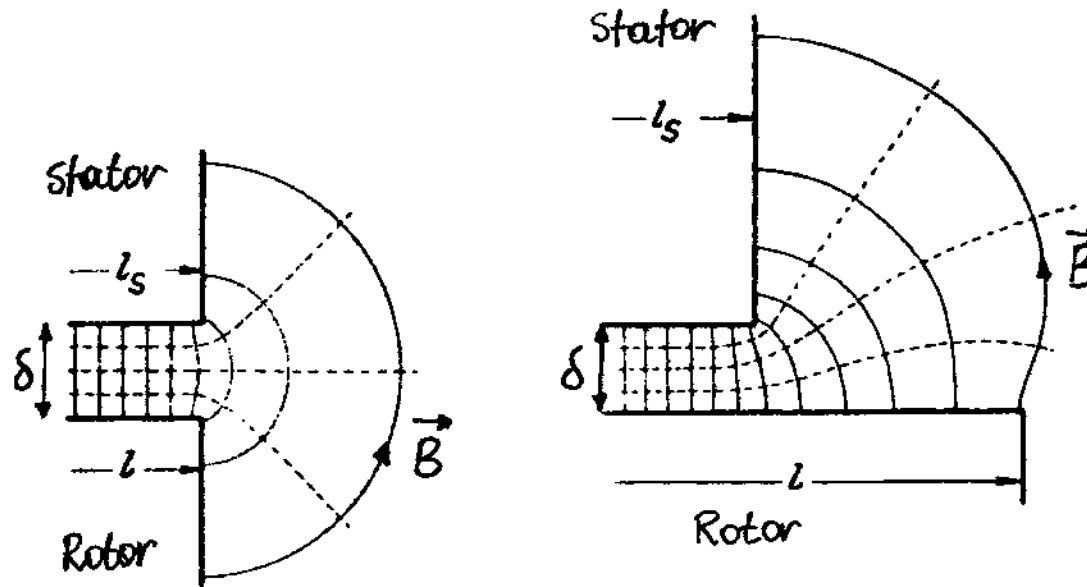
up to 1.8 GW)



# 3. Eddy current losses in winding systems

## Axial component of no-load end field

- Main flux density in air gap has end field component, penetrating press plates axially
- Big machines: big press plates = big surface for axial flux = big eddy currents



**Reducing losses:** - Partitioning of end plates, - slitting of press fingers, - Non-magnetic press plates to reduce magnetic field (Aluminium, stainless steel), -Stepping of end packet ( = increase of air gap) to reduce field, - laminated glued press plates, - Copper shielding

# 3. Eddy current losses in winding systems

End zone of synchronous hydro generator with high pole count



Source:  
VA Tech Hydro,  
Austria

# 3. Eddy current losses in winding systems

Stepping of end packet to increase air gap



Source:

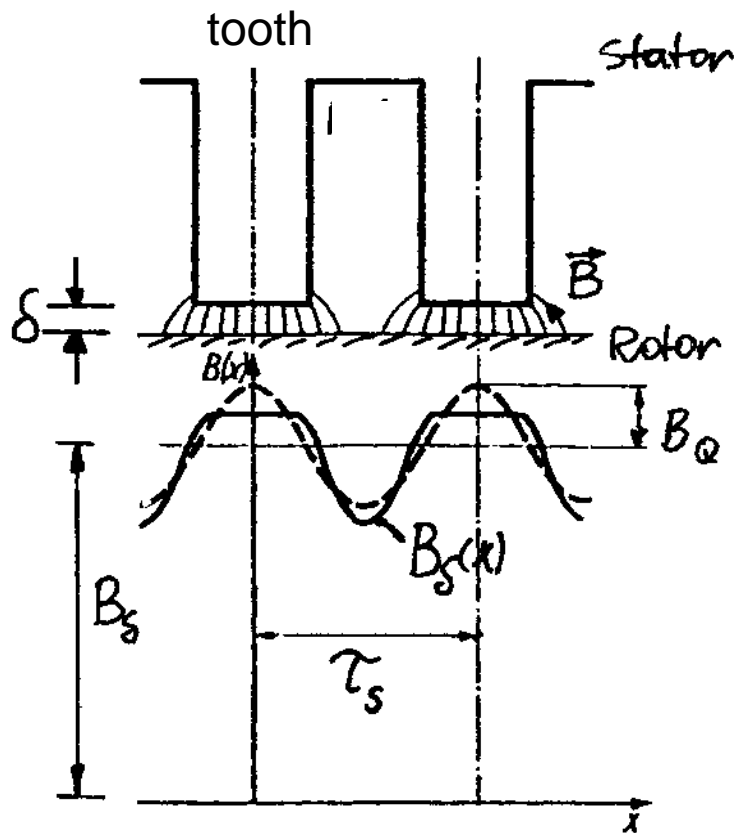
VATech Hydro,  
Austria



# 3. Eddy current losses in winding systems

## Rotor pole shoe losses

- High voltage winding demands open stator slots, which cause ripple in air gap magnetic flux density



- Slot-frequent ripple causes induced voltage in rotor pole shoe surface, hence eddy current losses occur !

- Frequency:  $f_Q = n \cdot Q$

(in kHz-range !: e.g. 36 slots, 3000/min:  
 $f_Q = 1800$  Hz)

- Reduction of pole shoe losses:

- Laminated pole shoes
- Magnetic stator slot wedges
- Grooving of rotor press plates

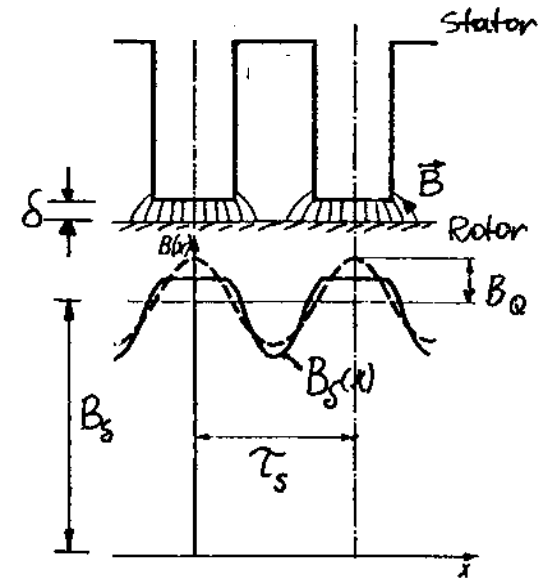
# 3. Eddy current losses in winding systems

## Manufacturing of poles for high pole count low speed ring generator



Pole shoes, built as laminated iron stack to suppress eddy currents, which are induced by slot ripple magnetic air gap field due to stator slot openings

Slots for damper bars



Source: VA Tech Hydro, Austria



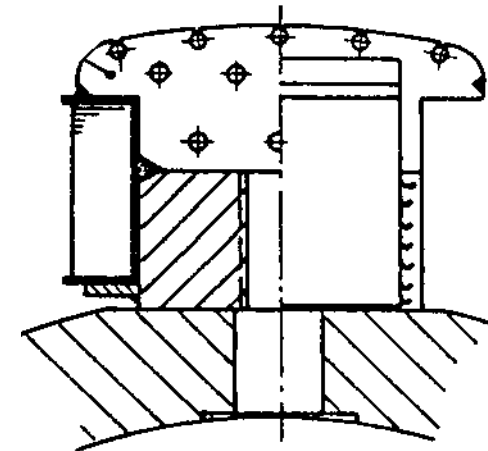
# 3. Eddy current losses in winding systems

## Massive rotor pole shaft welded to laminated pole shoes



Laminated pole shoes

Massive pole shaft (welded)



Source:

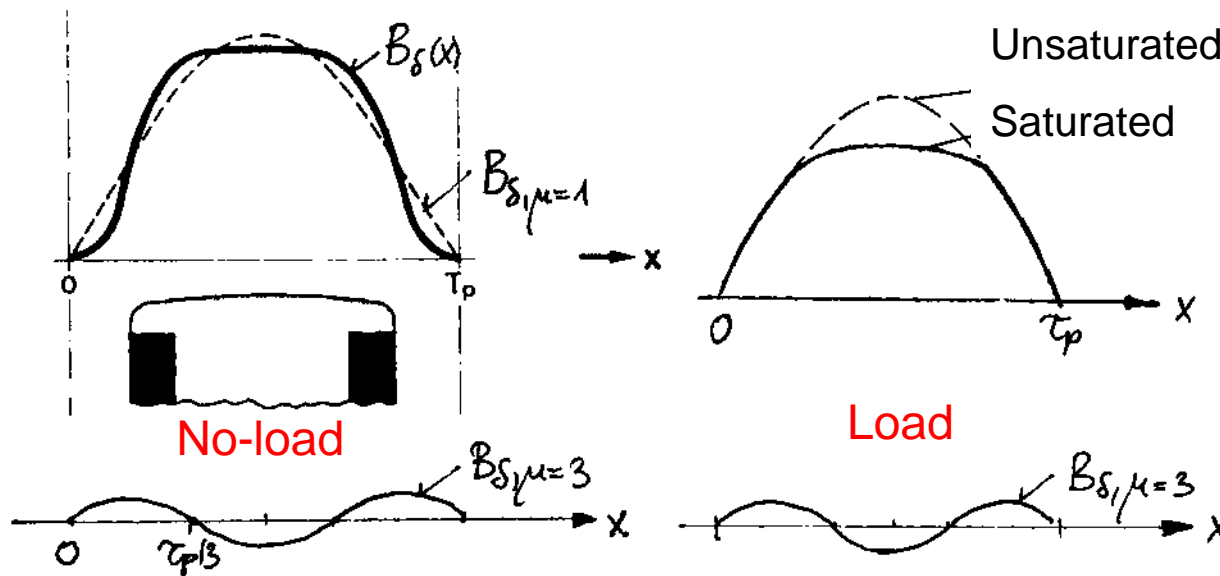
VATech Hydro,  
Austria

# 3. Eddy current losses in winding systems

## Additional load losses

- **Stator iron losses due to 3<sup>rd</sup> harmonic of air gap field**

- Already at **no-load** 3<sup>rd</sup> harmonic due to shape of air gap exists in salient pole machines !
- **At load** stator and rotor field give increased flux: Saturation occurs, causing increased 3<sup>rd</sup> harmonic



- 3<sup>rd</sup> harmonic stator frequency is 3-times line frequency

e.g. 150 Hz

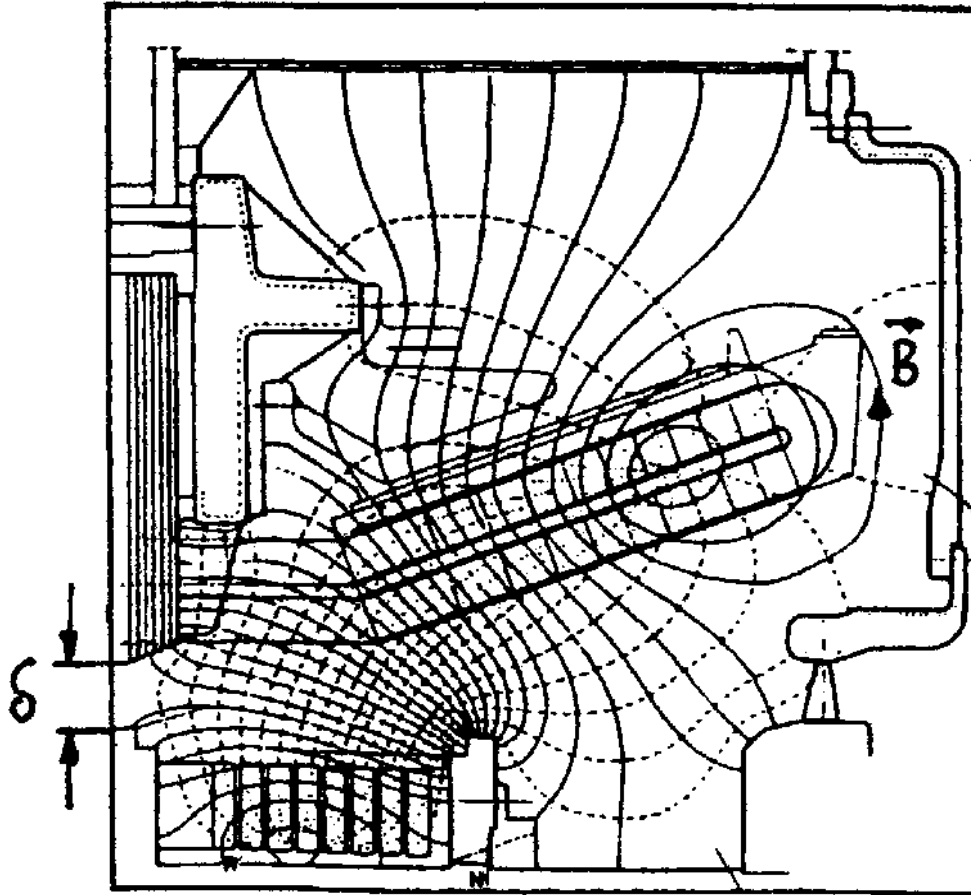
- Reduction of losses:

Optimization of air gap

Low loss iron sheets

# 3. Eddy current losses in winding systems

## Increase of press plate losses under load



- Winding overhang of stator and rotor winding excite stray field with line frequency
- Axial component causes considerable eddy current losses in press plates
- Reduction of losses:
  - Partitioning of end plates,
  - slitting of press fingers,
  - Non-magnetic press plates to reduce magnetic field (Aluminium, stainless steel),
  - Stepping of end packet ( = increase of air gap) to reduce field, - laminated glued press plates,
  - Copper shielding

# 3. Eddy current losses in winding systems

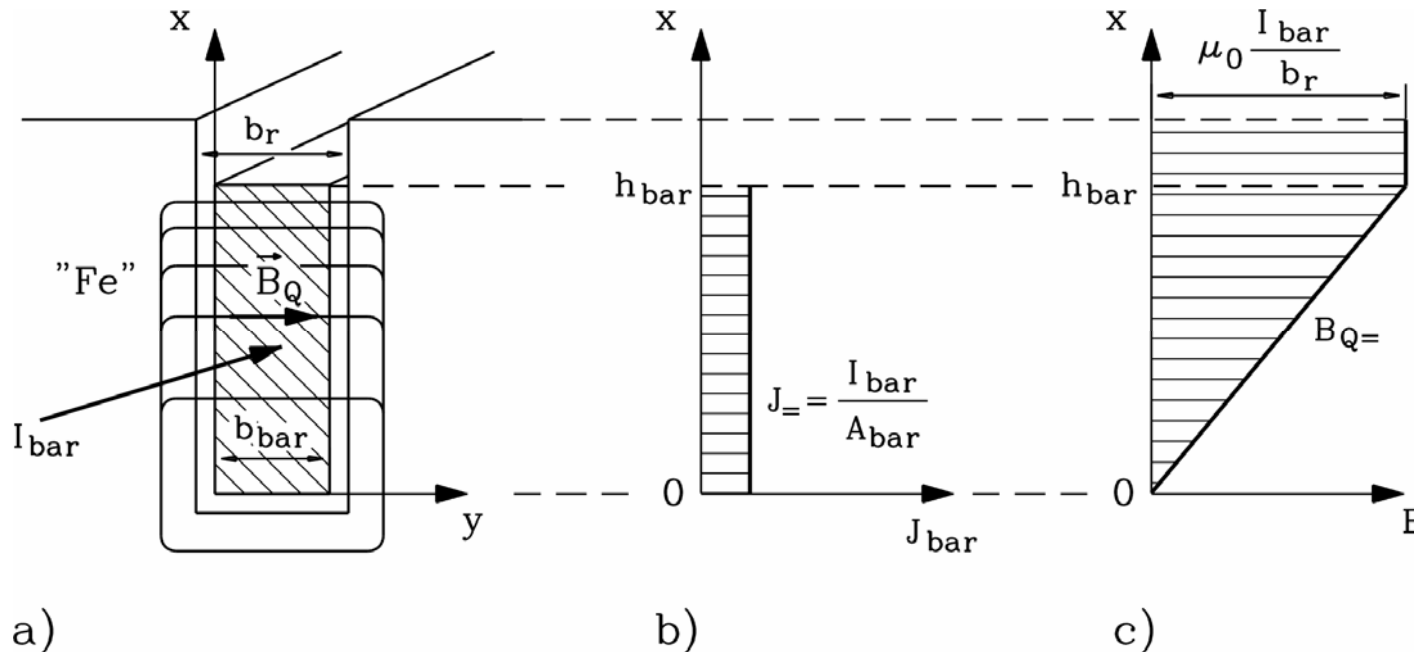
## 3.2 Basics on current displacement

- Big generators: Big power leads to **big current !**
- Big current needs big conductor cross section, so **conductor height is big !**
- Usually in big generators number of turns per coil is  $N_c = 1$ : = BAR !
- Stator current causes slot stray flux, pulsating with line frequency, which induces eddy currents in conductor itself = **additional load losses !**
- Eddy currents are superimposed on bar current; they cause uneven current density distribution ! Current flows to greater part in upper conductor half = **current displacement !**



# 3. Eddy current losses in winding systems

- If current density  $J_{bar} = I_{bar} / A_{bar}$  is homogeneously distributed over bar cross section, then **slot stray field**, which **crosses slot perpendicular** to slot axis, increases **linear** with bar height  $x$  !

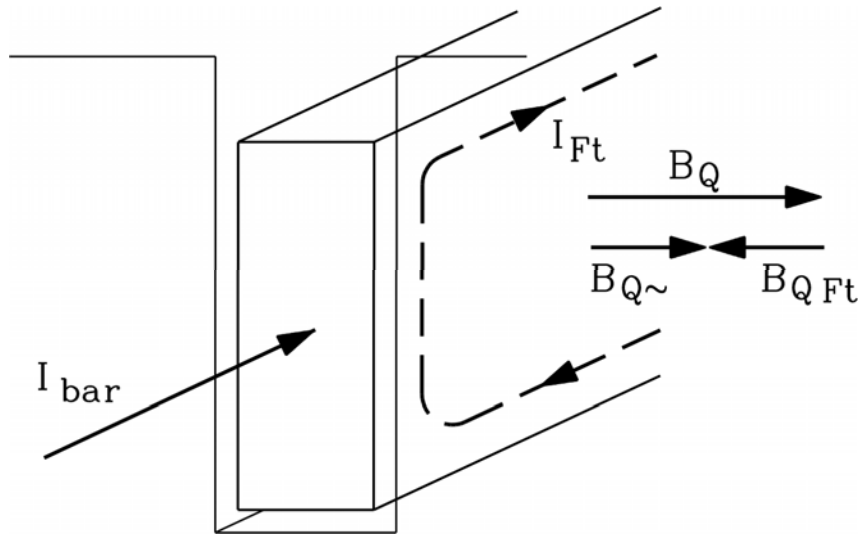


- **AMPERE´S law:**  $\oint_C \vec{H} \cdot d\vec{s} = H_Q(x) \cdot b_r = J \cdot x \cdot b_{bar}$

$$B_Q(x) = \mu_0 J \frac{x \cdot b_{bar}}{b_r} = \mu_0 \frac{I_{bar}}{b_r} \cdot \frac{x}{h_{bar}}, 0 \leq x \leq h_{bar} \quad \text{or} \quad B_Q = \mu_0 \frac{I_{bar}}{b_r}, x > h_{bar}$$

# 3. Eddy current losses in winding systems

- **Slot flux density** is pulsating with line frequency, penetrating the slot bar from the side. High slot bars form a "massive short circuit loop". **FARADAY's** law yields:  $B_Q$  induces voltage  $u_j = -d\Phi/dt$  in bar, which causes **eddy current flow**  $I_{Ft}$ . Self field of that eddy current  $B_{Q_{Ft}}$  is directed opposite to  $B_Q$  due to **LENZ's rule**.
- Hence the **eddy current**  $I_{Ft}$  flows in upper bar region IN direction of bar current  $I_{bar}$ , and in lower bar region OPPOSITE to bar current.



• **Facit 1:**

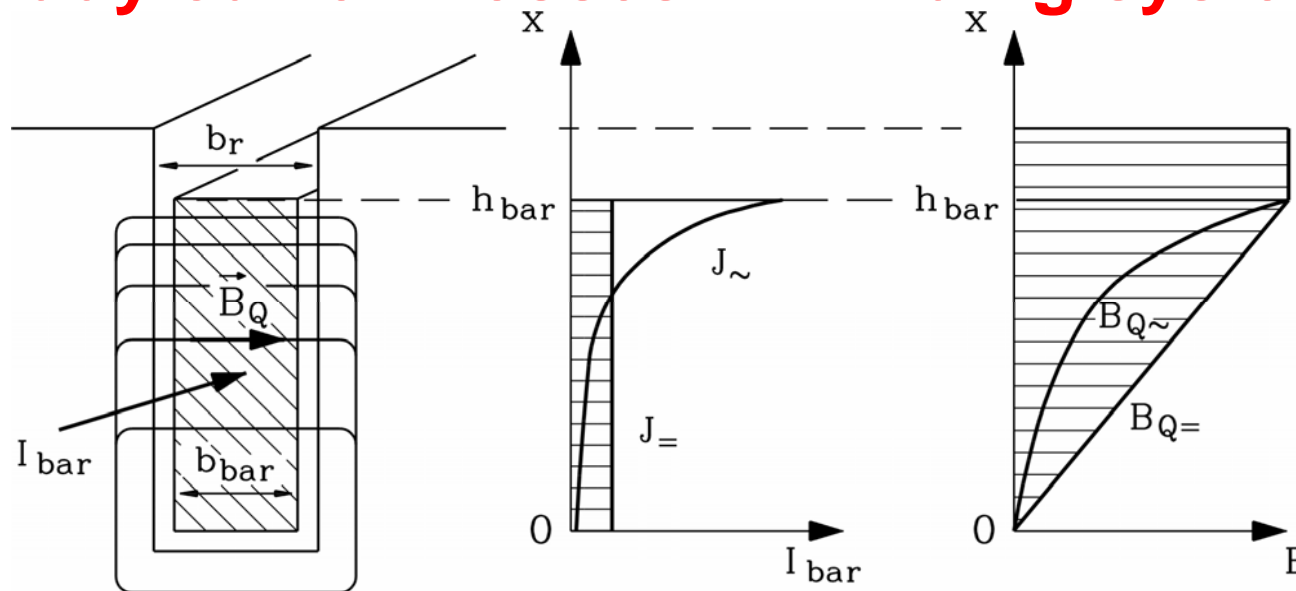
Due to  $I_{Ft}$  the resulting bar current density is **HIGHER** in upper bar region: **Current displacement towards upper bar region ("Skin effect")**.

• **Facit 2:**

The **resulting** slot stray flux density  $B_{Q\sim}$  is due to  $B_{Q_{Ft}}$  **reduced**.

- **Current displacement INCREASES** with increasing frequency  $f_r$ , with increasing electric bar-conductivity  $\kappa$ , with increasing bar height  $h_{bar}$  and with increasing permeability  $\mu$  of conductor. (**Note:** Copper and aluminium's permeability is  $\mu = \mu_0$  !)

### 3. Eddy current losses in winding systems



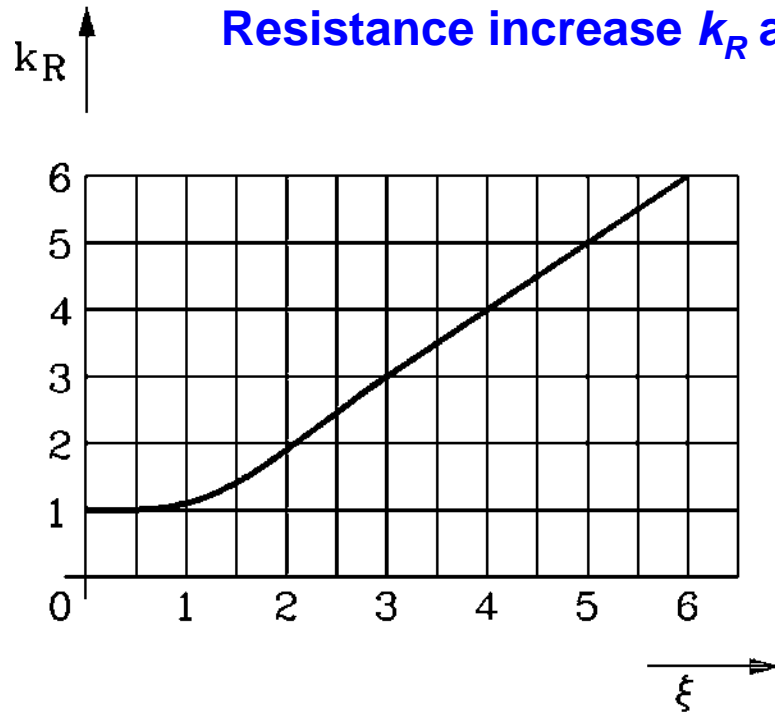
- Due to high stator frequency major part of bar current flows in upper bar region: so only reduced bar cross section is used for current flow. Thus “AC bar resistance”  $R_{bar\sim}$  is higher than “DC bar resistance”  $R_{bar=}$

- Due to reduction of slot stray flux density the slot leakage flux is reduced. Hence the “AC bar inductance”  $L_{bar\sim}$  is smaller than the “DC bar inductance”  $L_{bar=}$

$$R_{bar\sim} = k_R R_{bar=} > R_{bar=}$$

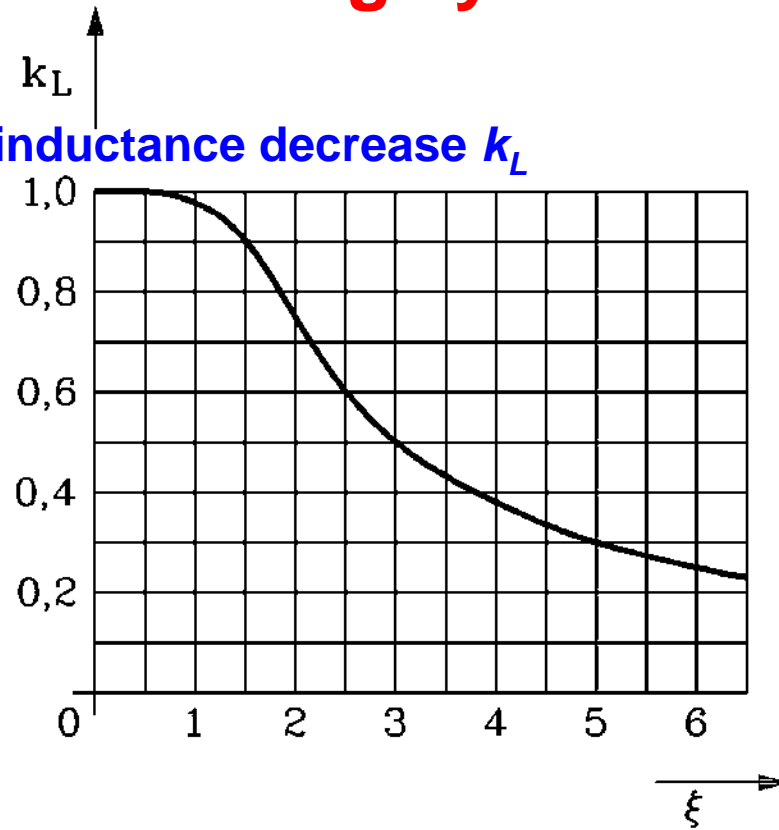
$$L_{\sigma,bar\sim} = k_L L_{\sigma,bar=} < L_{\sigma,bar=}$$

# 3. Eddy current losses in winding systems



a)

Resistance increase  $k_R$  and inductance decrease  $k_L$



b)

- “Reduced” conductor height  $\xi$  : Per-unit value  $\xi$ , containing all relevant parameters:

$k_R, k_L$  for deep bar (“rectangular cross section”) depend on:

$$\xi = h_{bar} \sqrt{\pi f_s \mu \kappa \frac{b_{bar}}{b_s}}$$



# 3. Eddy current losses in winding systems

- **Example: Current displacement in slot bar : Copper bar:**

- At 75°C bar temperature copper conductivity is  $\kappa_{Cu} = 50 \cdot 10^6 \text{ S/m}$ .
- Bar width = slot width:  $b_{bar} = b_s$ ,
- Permeability:  $\mu_{Cu} = \mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/(Am)}$
- Stator frequency  $f_s = 50 \text{ Hz}$
- Bar height:  $h_{bar} = 3 \text{ cm}$

$$\xi = h_{bar} \sqrt{\pi f_s \mu \kappa \frac{b_{bar}}{b_s}} = 3 \cdot 10^{-2} \cdot \sqrt{\pi \cdot 50 \cdot 4\pi \cdot 10^{-7} \cdot 50 \cdot 10^6 \cdot 1} = 2.98 \approx 3$$

From curve  $k_R(\xi)$  we get:  $k_R(3) = 3$  and from  $k_L(\xi)$  follows:  $k_L(3) = 0.5$ .

- **Facit:**

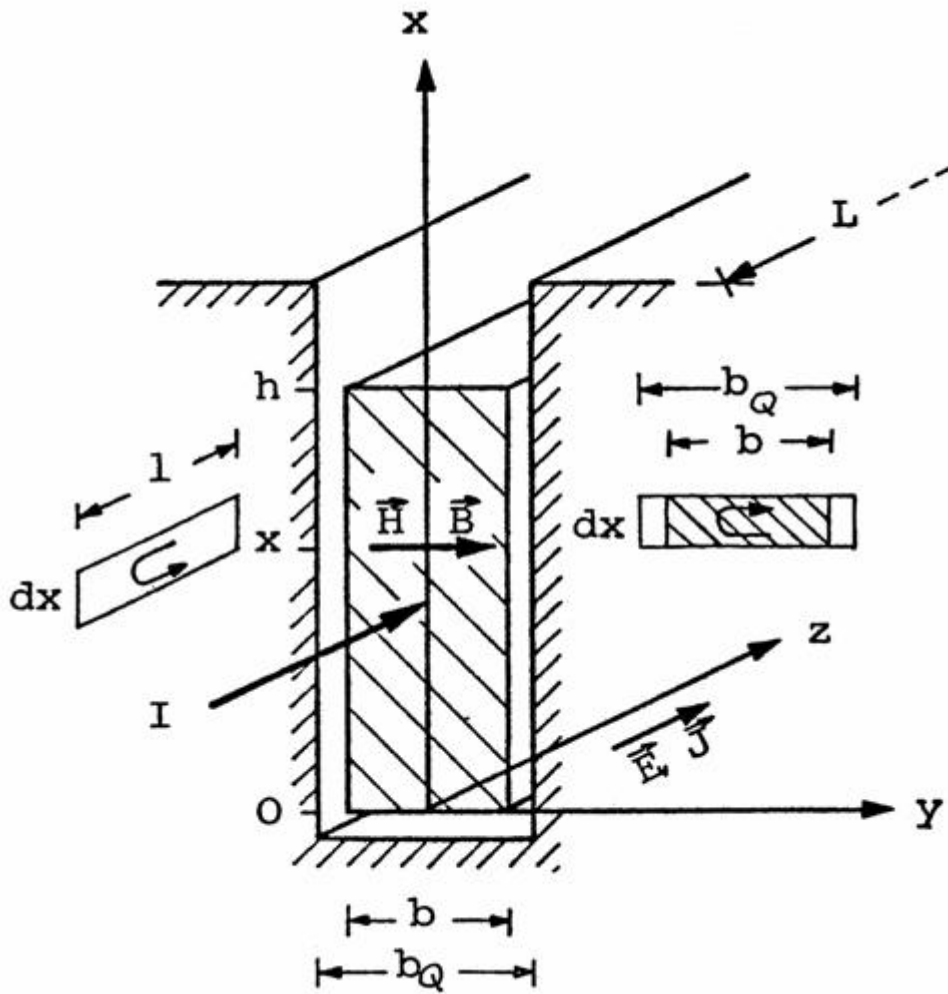
- Bar resistance increases up to **3-fold !**
- Bar inductance **decreases down to 50%**.

- **Thumb rule:**

At 50 Hz the increase of resistance of copper deep bar is  $k_R = h_{bar} [cm]$  .

# 3. Eddy current losses in winding systems

## 3.3 Current displacement in massive slot conductor



- **Ampere's law:**

$$\oint_C \vec{H} \cdot d\vec{s} = \Theta = \vec{J} \cdot \vec{A} = \kappa \vec{E} \cdot \vec{A}$$

$$-H_y(x)b_Q + H_y(x+dx)b_Q = \kappa E_z b dx$$

$$-H_y(x)b_Q + \left(H_y(x) + \frac{\partial H_y}{\partial x} dx\right)b_Q = \kappa E_z b dx$$

$$\frac{\partial H_y}{\partial x} = \kappa \frac{b}{b_Q} E_z$$

**1. Maxwell equation**

- **Faraday's law:**

$$\oint_C \vec{E} \cdot d\vec{s} = u_i = -\frac{\partial \Phi}{\partial t}$$

$$E_z(x)l - E_z(x+dx)l = -\mu l \cdot dx \cdot \frac{\partial H_y}{\partial t}$$

$$E_z(x)l - \left(E_z(x) + \frac{\partial E_z}{\partial x} dx\right)l = -\mu l \cdot dx \cdot \frac{\partial H_y}{\partial t}$$

$$\frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t}$$

**2. Maxwell equation**

# 3. Eddy current losses in winding systems

## Calculation of eddy currents in slot conductor (1)

- Solution of linear partial differential equations for sinusoidal time functions: Use of complex phasors: yields conventional linear differential equation!

$$H(x, t) = \text{Re}\{\underline{H}(x) \cdot \sqrt{2} \cdot e^{j\omega t}\}, \quad E(x, t) = \text{Re}\{\underline{E}(x) \cdot \sqrt{2} \cdot e^{j\omega t}\}, \quad J(x, t) = \text{Re}\{\underline{J}(x) \cdot \sqrt{2} \cdot e^{j\omega t}\}$$

$$H(x, t) \rightarrow \underline{H}(x), \quad E(x, t) \rightarrow \underline{E}(x), \quad J(x, t) \rightarrow \underline{J}(x)$$

$$\frac{\partial \underline{H}_y}{\partial x} = \kappa \frac{b}{b_Q} \underline{E}_z \quad \frac{\partial \underline{E}_z}{\partial x} = j\omega\mu \underline{H}_y \quad \longrightarrow \quad \boxed{\frac{d^2 \underline{E}_z(x)}{dx^2} - j\omega\mu\kappa \frac{b}{b_Q} \underline{E}_z(x) = 0}$$

- Solution of conventional linear differential equation: exponential functions !

$$\underline{E}_z(x) = \underline{C} e^{\lambda x} \quad \lambda_{1,2} = \pm \sqrt{j\omega\mu\kappa \frac{b}{b_Q}} = \pm(1+j) \sqrt{\pi f \mu \kappa \frac{b}{b_Q}} = \pm(1+j)\beta$$

$$\boxed{\underline{E}_z(x) = \underline{C}_1 e^{-(1+j)\beta x} + \underline{C}_2 e^{(1+j)\beta x}}$$

$$\beta = \sqrt{\pi f \mu \kappa \frac{b}{b_Q}}$$

### 3. Eddy current losses in winding systems

#### Calculation of eddy currents in slot conductor (2)

$$\frac{\partial \underline{E}_z}{\partial x} = j\omega\mu \underline{H}_y \quad \longrightarrow \quad \underline{H}_y(x) = -\frac{1+j}{j} \cdot \frac{\beta}{\omega\mu} \left( \underline{C}_1 e^{-(1+j)\beta x} - \underline{C}_2 e^{(1+j)\beta x} \right)$$

#### Boundary conditions:

At lower bar edge ( $x = 0$ ) magnetic field is zero:  $\underline{H}_y(0) = 0$ :  $\underline{C}_1 = \underline{C}_2 = \underline{C}$ .

$$\underline{H}_y(x) = \frac{1+j}{j} \cdot \frac{\beta}{\omega\mu} \cdot \underline{C} \left( -e^{-(1+j)\beta x} + e^{(1+j)\beta x} \right) = \frac{1+j}{j} \cdot \frac{\beta}{\omega\mu} \cdot \underline{C} \cdot 2 \cdot \text{sh}[(1+j)\beta x]$$

At upper bar edge  $x = h$  the m.m.f. is equal to bar current  $I$ ; so magnetic field is:

$$\underline{H}_y(x=h) = I/b_Q. \quad \underline{C} = \frac{j}{1+j} \cdot \frac{\omega\mu}{\beta b_Q} \cdot \frac{I}{2 \cdot \text{sh}[(1+j)\beta h]}$$

$$\underline{H}_y(x) = \frac{I}{b_Q} \cdot \frac{\text{sh}[(1+j)\beta x]}{\text{sh}[(1+j)\beta h]}$$

$$\underline{E}_z(x) = \frac{j}{1+j} \cdot \frac{\omega\mu}{\beta} \cdot \frac{I}{b_Q} \cdot \frac{\text{ch}[(1+j)\beta x]}{\text{sh}[(1+j)\beta h]}$$

$$\underline{J}_z(x) = \kappa \underline{E}_z = \frac{j}{1+j} \cdot \frac{\omega\mu\kappa}{\beta} \cdot \frac{I}{b_Q} \cdot \frac{\text{ch}[(1+j)\beta x]}{\text{sh}[(1+j)\beta h]}$$

# 3. Eddy current losses in winding systems

## Current density distribution

- **Absolute value** of current density distribution  $J$ :

Use formulas:  $\text{sh}(x + jy) = \text{sh } x \cos y + j \text{ch } x \sin y$   
 $\text{ch}(x + jy) = \text{ch } x \cos y + j \text{sh } x \sin y$

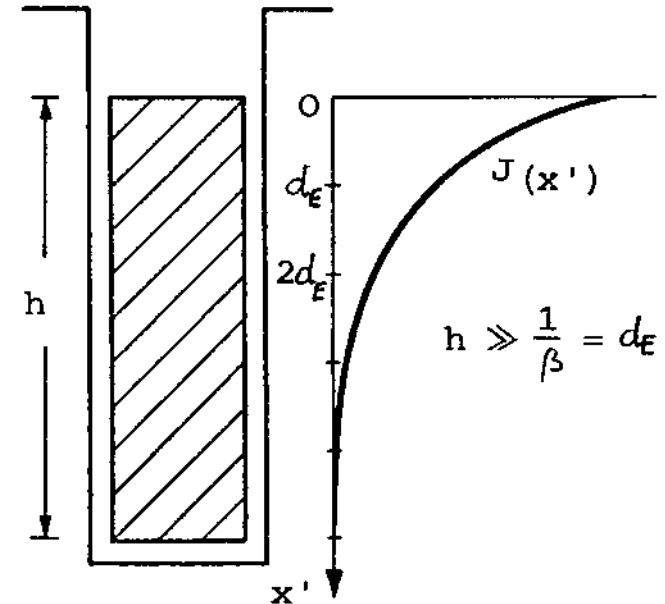
$$J_z(x) = \frac{1}{\sqrt{2}} \cdot \frac{\omega \mu \kappa}{\beta} \cdot \frac{I}{b_Q} \cdot \sqrt{\frac{\text{ch} 2\beta x + \cos 2\beta x}{\text{ch} 2\beta h - \cos 2\beta h}}$$

- **Simplified expression** for big  $\beta$ :  $h \gg 1/\beta$ :  
 $\cos(2\beta h) \ll \text{ch}(2\beta h)$ ,  $x' = h - x$ ,

$$\text{ch}(2\beta \cdot x) = (e^{2\beta \cdot x} + e^{-2\beta \cdot x}) / 2 \approx e^{2\beta \cdot x} / 2$$

$$\sqrt{\text{ch}(2\beta \cdot x) / \text{ch}(2\beta \cdot h)} \approx \sqrt{e^{2\beta \cdot x} / e^{2\beta \cdot h}} = e^{\beta(x-h)} = e^{-\beta \cdot x'}$$

$$J_z(x') = \frac{1}{\sqrt{2}} \cdot \frac{\omega \mu \kappa}{\beta} \cdot \frac{I}{b_Q} \cdot e^{-\beta \cdot x'}$$



**Penetration depth  $d_E$**

$$d_E = \frac{1}{\beta} = \sqrt{\frac{b_Q}{b} \cdot \frac{1}{\pi f \mu \kappa}}$$

# 3. Eddy current losses in winding systems

## Increased losses due to eddy currents

- **Total losses** in conductor with length  $L$ :  $P_1 = \frac{b \cdot L}{\kappa} \cdot \int_0^h J_z^2 dx = I^2 R_{\sim} = k_R \cdot I^2 R_0$
- **DC resistance** of conductor:  $R_0 = \frac{L}{bh\kappa}$

$$P_1 = \frac{b \cdot L}{\kappa} \cdot \int_0^h \frac{1}{2} \left( \frac{\omega \mu \kappa}{\beta} \right)^2 \cdot \left( \frac{I}{b_Q} \right)^2 \cdot \frac{\cosh(2\beta x) + \cos(2\beta x)}{\cosh(2\beta h) - \cos(2\beta h)} dx = I^2 \cdot R_0 \cdot \xi \cdot \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}$$

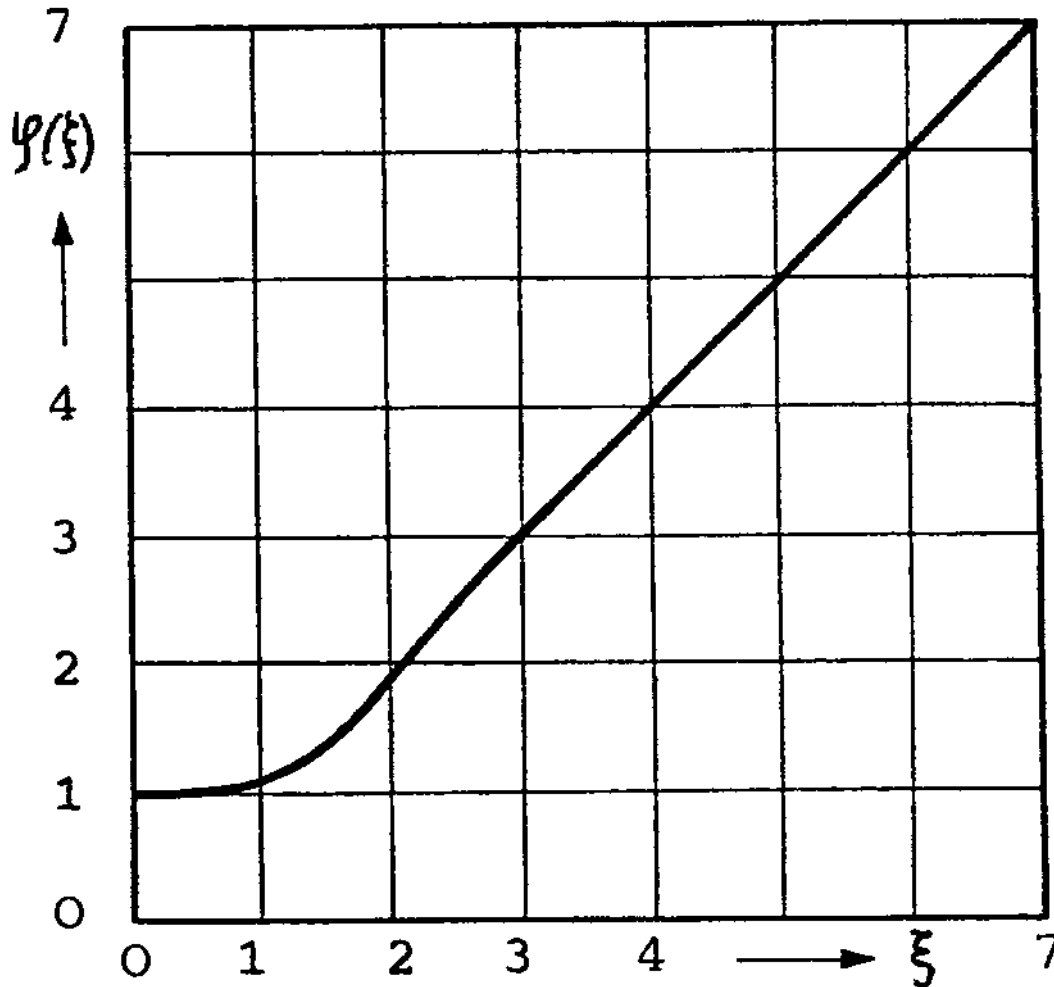
- **Increased losses** by coefficient:  $k_R = \frac{R_{\sim}}{R_0} = \varphi(\xi) = \xi \cdot \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}$

Abbreviation: "Reduced" conductor height:  $\xi = \beta \cdot h = \frac{h}{d_E} = h \cdot \sqrt{\pi f \mu \kappa \frac{b}{b_Q}}$

- Above  $h > 0.5 d_E$  current displacement is significant, rising above  $\xi = 3$  with nearly linear increase  $k_R \cong \xi$ .

### 3. Eddy current losses in winding systems

#### Increased losses in massive slot conductor



$$k_R = \frac{R_{\sim}}{R_0} = \varphi(\xi) = \xi \cdot \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}$$

$\xi > 3 :$

$$R_{\sim} = k_R R_0 \approx \xi R_0 = \frac{h}{d_E} \cdot \frac{L}{bh\kappa} = \frac{L}{bd_E\kappa}$$

# 3. Eddy current losses in winding systems

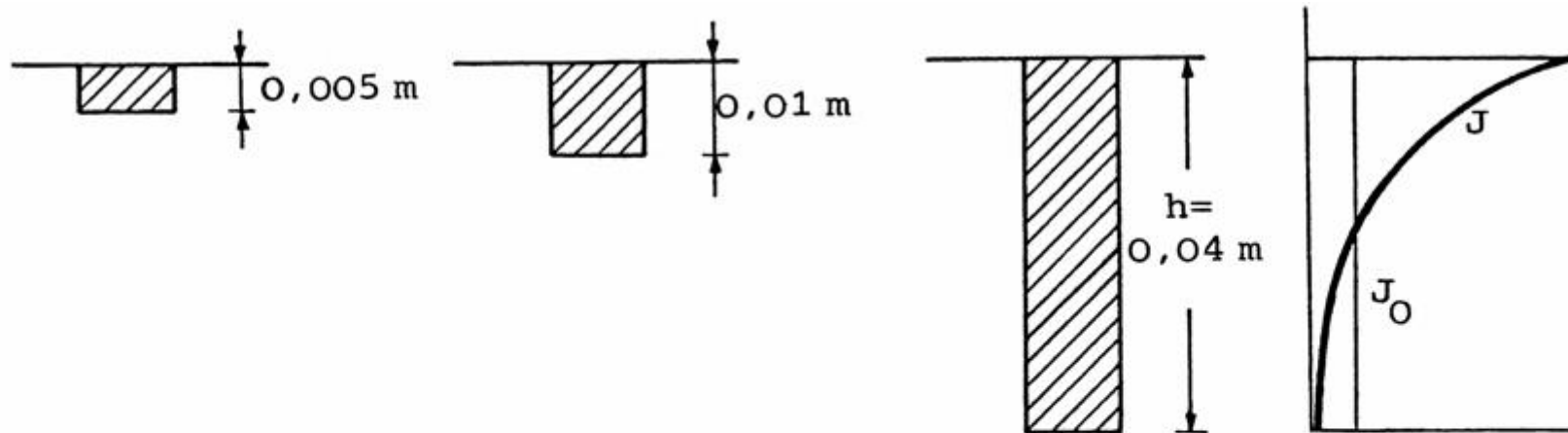
## Effect of increased losses for copper bar

Example:

Cu-bar with different height  $h$  at  $f = 50$  Hz:

$h$ : a) 5 mm, b) 10 mm, c) 40 mm

$\kappa = 50 \cdot 10^6$  1/m,  $\mu = \mu_0 = 4 \cdot \pi \cdot 10^{-7}$  Vs/(Am),  $b/b_Q = 1$ ,  $d_E = 0.01$  m = 10 mm



$$\xi = \frac{0,005}{0,01} = 0,5$$

$$k \approx 1,0$$

$$\frac{0,01}{0,01} = 1,0$$

$$k = 1,09$$

$$\frac{0,04}{0,01} = 4$$

$$k = 4,0$$

**Increase of losses**



# 3. Eddy current losses in winding systems

## Instantaneous current density distribution (1)

At different instants  $t$  : Current changes sinusoidal:  $i(t) = \sqrt{2} \cdot I \cdot \cos \omega t$  :

$$\underline{J}_z(x) = \kappa E_z = \frac{j}{1+j} \cdot \frac{\omega \mu \kappa}{\beta} \cdot \frac{I}{b_Q} \cdot \frac{ch[(1+j)\beta x]}{sh[(1+j)\beta h]} = J_{re}(x) + jJ_{im}(x)$$

$$J_z(x, t) = \text{Re} \left\{ \underline{J}_z(x) \cdot \sqrt{2} \cdot e^{j\omega t} \right\} = \sqrt{2} \cdot (J_{re}(x) \cdot \cos \omega t - J_{im}(x) \cdot \sin \omega t)$$

### Example:

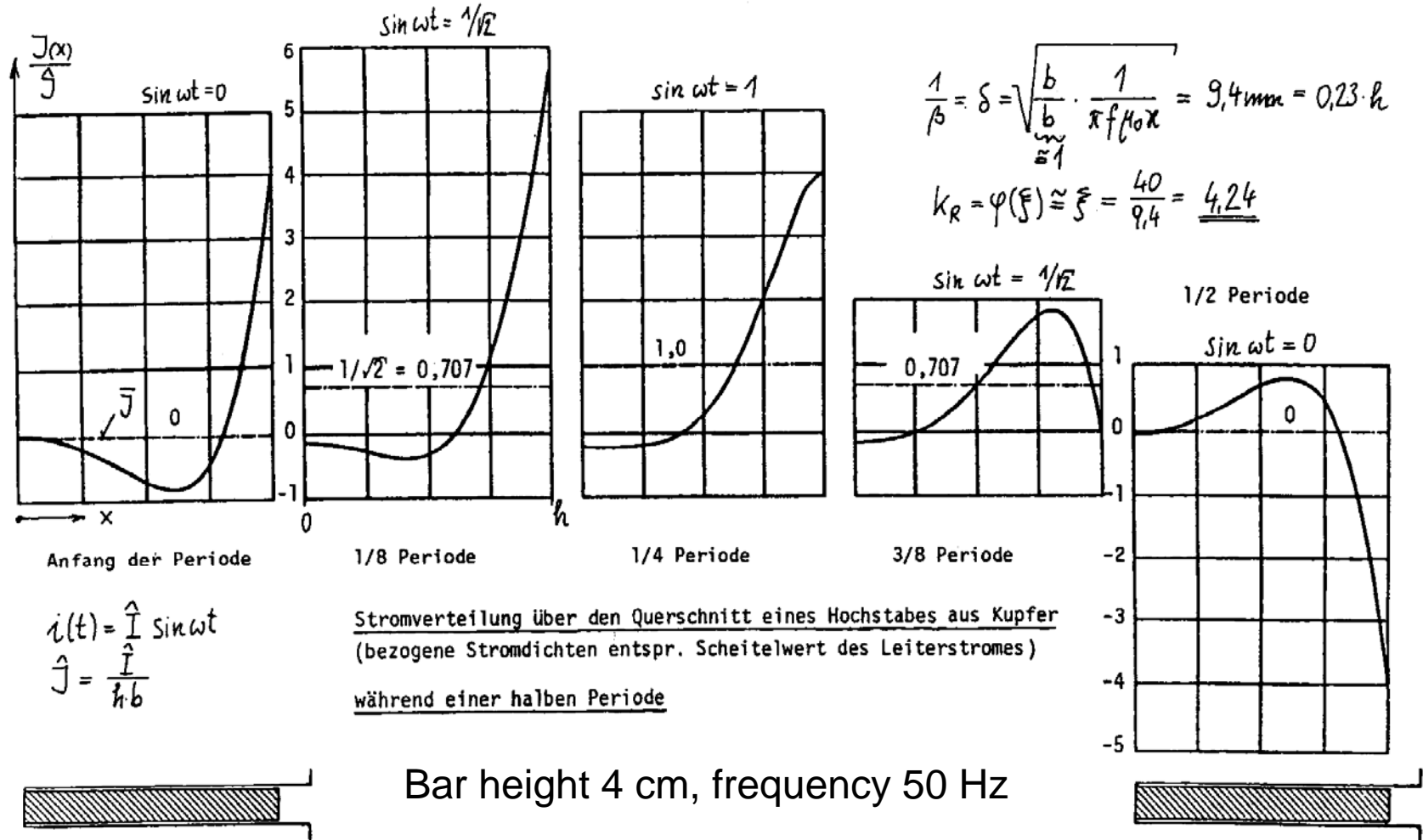
Copper bar,  $h = 4$  cm,  $f = 50$  Hz,  $b/b_Q = 1$ ,  $\kappa = 50 \cdot 10^6$  S/m,  $d_E = 1/\beta = 10$  mm,  $\xi = 4.0$ .

Current density  $J_z(x, t)$  for  $0 \leq x \leq h$  : per unit of DC current density  $J = I/(bh)$  :

Begin of period	$i = 0$	$\omega t = -\pi / 2$
1/8 period	$i = \hat{I} / \sqrt{2}$	$\omega t = -\pi / 4$
1/4 period	$i = \hat{I}$	$\omega t = 0$
3/8 period	$i = \hat{I} / \sqrt{2}$	$\omega t = \pi / 4$
Half period	$i = 0$	$\omega t = \pi / 2$

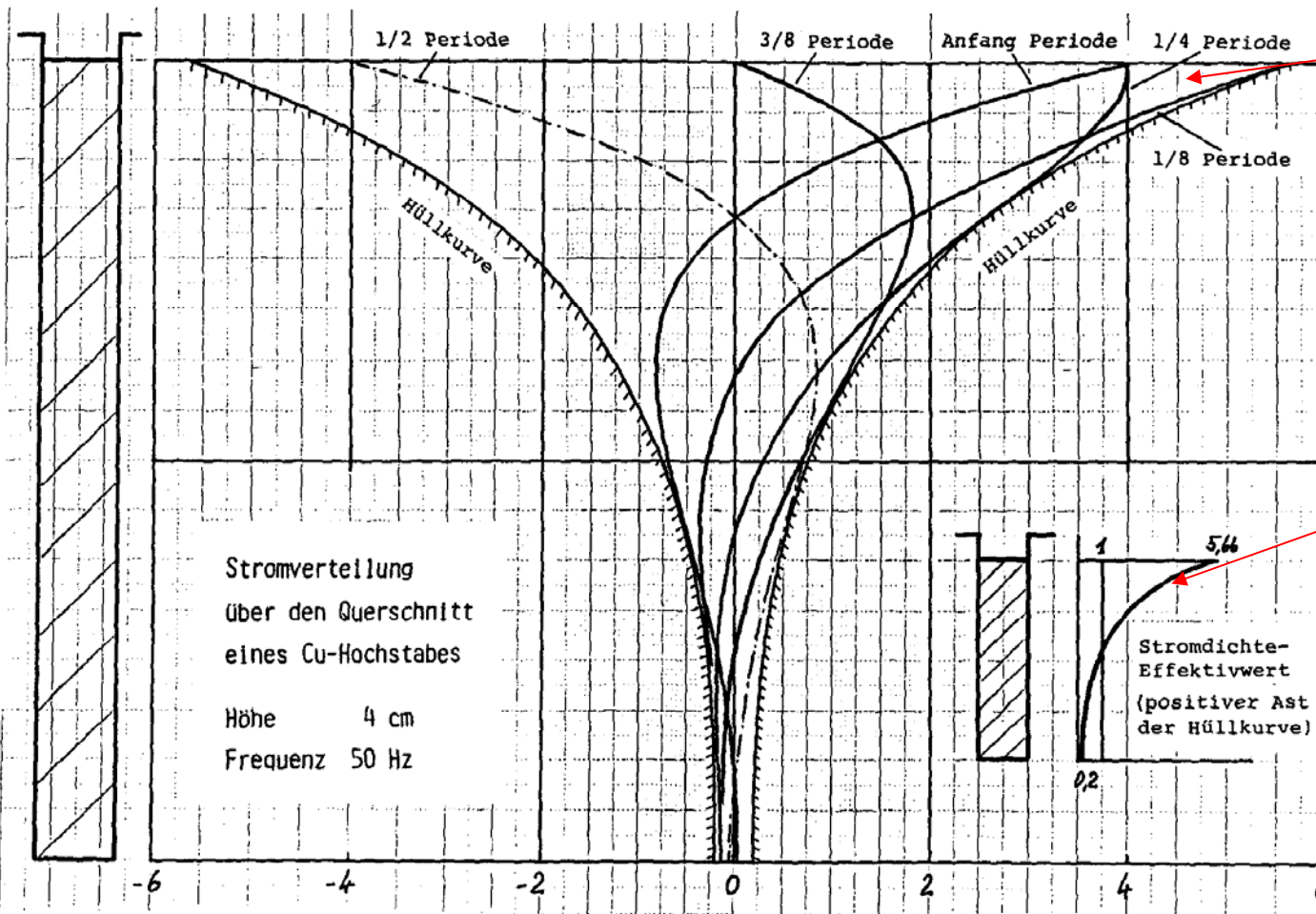
# 3. Eddy current losses in winding systems

## Instantaneous current density distribution (2)



# 3. Eddy current losses in winding systems

## Instantaneous current density distribution (3)



- Current density distribution along deep bar for 0, 1/8, 1/4, 3/8, 1/2 period
- Copper bar
- bar height 4 cm, frequency 50 Hz,
- The envelope of all instantaneous current density distribution is the resulting absolute value of current density distribution

# 3. Eddy current losses in winding systems

## Losses in massive conductive parts (1)

- If tangential AC field strength  $H_t$  at surface of body is given, with big body thickness  $h \gg \beta$  (e.g. thick plate), we get from previous solution:

$$J_z = \frac{1}{\sqrt{2}} \cdot \frac{\omega \mu \kappa}{\beta} \cdot \frac{I}{b_Q} \cdot e^{-\beta \cdot x'} = \frac{H_t}{d_E} \cdot e^{-x'/d_E}$$

- Losses : (e.g. in thick plate):  $P_1 = A \cdot \int_0^{\infty} \frac{J_z^2}{\kappa} \cdot dx' = A \cdot \frac{H_t^2}{2} \cdot \sqrt{\frac{\pi f \mu}{\kappa}}$

Losses per surface e.g. of plate:

$$\frac{P_1}{A} = \frac{H_t^2}{2\kappa} \cdot \frac{1}{d_E}$$

- **Losses are small:** a) if penetration depth is big,  
b) if conductivity is big.

### 3. Eddy current losses in winding systems

#### Losses in massive conductive parts (2)

<i>Material</i>	$\kappa / \text{S/m}$	$d_E / \text{mm}$	$P_1/A$ (per unit losses)
Copper	$50 \cdot 10^6$	10	1.0
Aluminium	$29 \cdot 10^6$	13	1.32
Iron, steel ( $\mu_r = 100$ )	$4 \cdot 10^6$	3.5	35.7
Iron, steel ( $\mu_r = 1000$ )	$4 \cdot 10^6$	1.1	113.6
Non-magnetic steel ( $\mu_r = 1$ )	$1.5 \cdot 10^6$	57.5	5.8

**Penetration depth and eddy current losses at  $f = 50 \text{ Hz}$ ,  $55^\circ\text{C}$  !**

#### Application:

- Copper plate for electro-dynamic press plate shielding.
- Aluminium or non-magnetic steel press plates instead of magnetic iron press plates

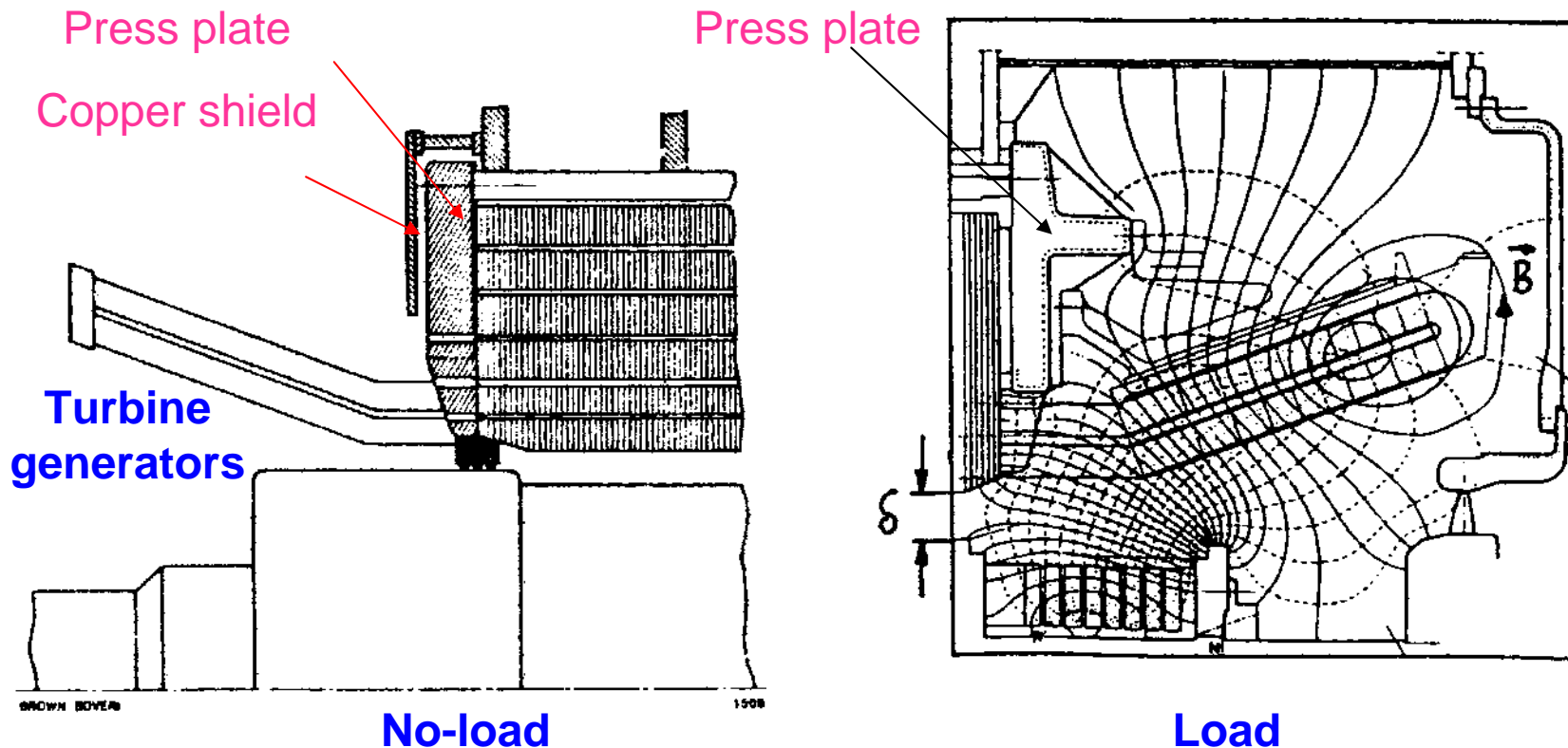


# 3. Eddy current losses in winding systems

## Press plate losses

- Losses in press plates and press fingers:

Axial end field component induces with stator frequency eddy currents in press plates



# 3. Eddy current losses in winding systems

## 3.4 Critical conductor height

a) Ohmic losses:

$$P_0 = I^2 R_0 = \frac{I^2 L}{bkh} = \frac{I^2 L}{bd_E K} \cdot \frac{d_E}{h} = \frac{P_0 d_E}{\xi}$$

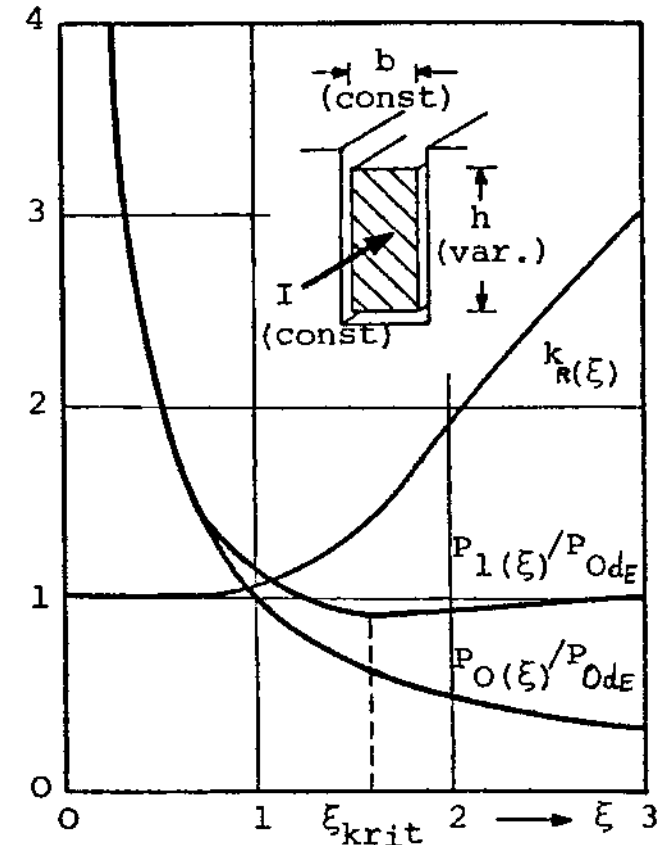
b) AC losses:

$$P_1 = I^2 R_{\sim} = I^2 R_0 \frac{R_{\sim}}{R_0} = P_0 k_R = P_0 d_E \cdot \frac{sh(2\xi) + \sin(2\xi)}{ch(2\xi) - \cos(2\xi)}$$

- With increasing conductor height at given current a) decreases, but b) increases. **Where is optimum?** Differentiation:

$dP_1/d\xi = 0$  yields  $sh(2\xi) \cdot \sin(2\xi) = 0$  at

$$\xi_{crit} = \frac{\pi}{2} = 1.57$$



# 3. Eddy current losses in winding systems

## Minimum “massive” bar conductor losses

- Minimum total losses at critical conductor height: optimum coefficient:

$$k_{R,opt} = \varphi(\xi_{crit}) = \varphi(\pi / 2) = 1.44$$

Example:

Cu-conductor,  $f = 50$  Hz:  $\Rightarrow d_E = 0.01$  m:  $h_{krit} = 0.0157$  m.

- How big are total losses for infinite conductor height ? **FINITE !**

$$P_1(\xi \rightarrow \infty) \approx P_{0d_E} \cdot th(2\xi) = P_{0d_E}$$

$$P_{0d_E} = \frac{I^2 L}{bd_E \kappa}$$

*Facit:*

For big machines (big current = big conductor cross section) it is necessary to split conductor into smaller strands to avoid excessive AC losses !

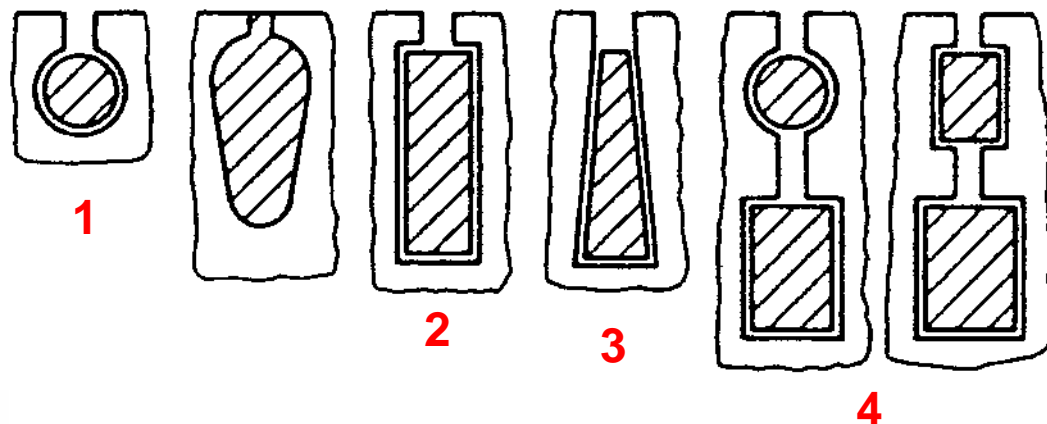
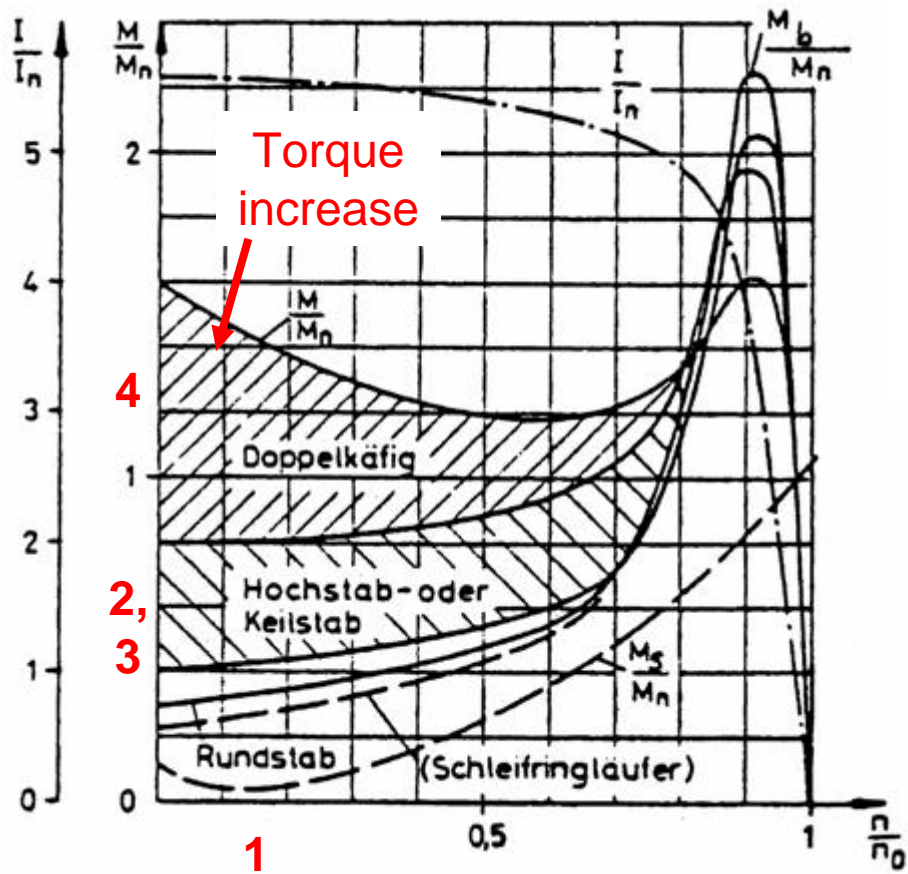




# 3. Eddy current losses in winding systems

## 3.5 Use of current displacement in electrical machines

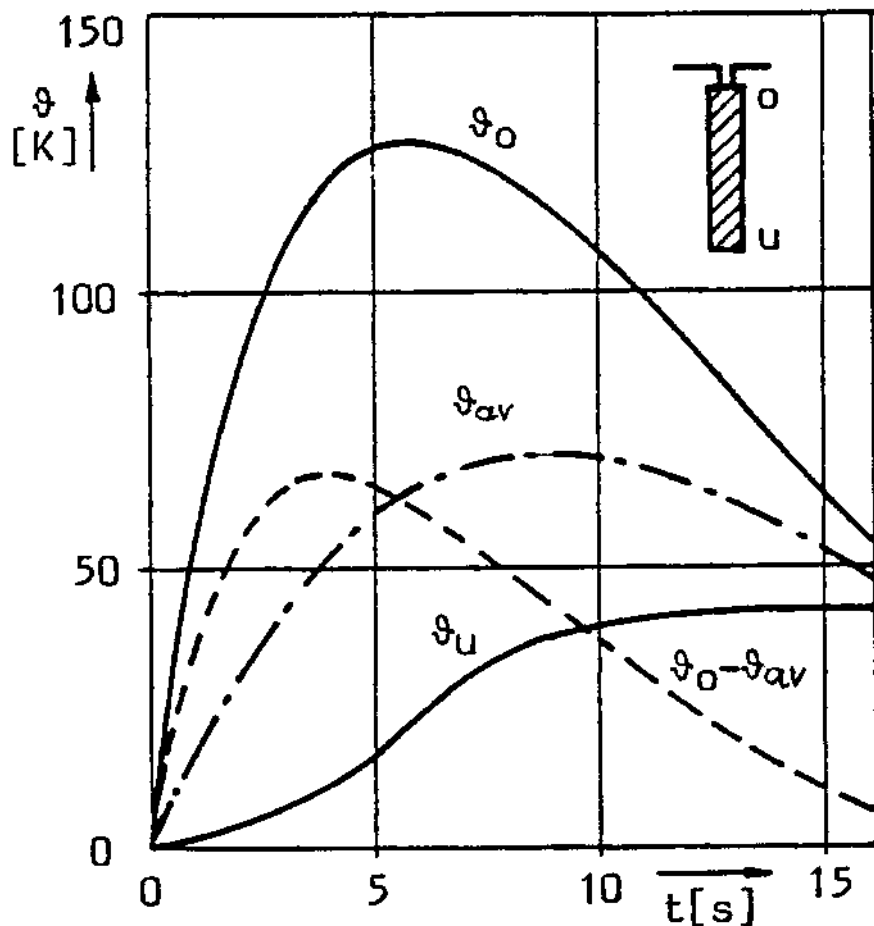
Increase of asynchronous starting torque in induction machines



- Asynchronous starting torque due to rotor cage is directly proportional to rotor losses.
- So at stand still = big rotor frequency - big AC losses (with bar shapes 2, 3, 4) yield big torque increase !

# 3. Eddy current losses in winding systems

## Heating of rotor bar during asynchronous start-up

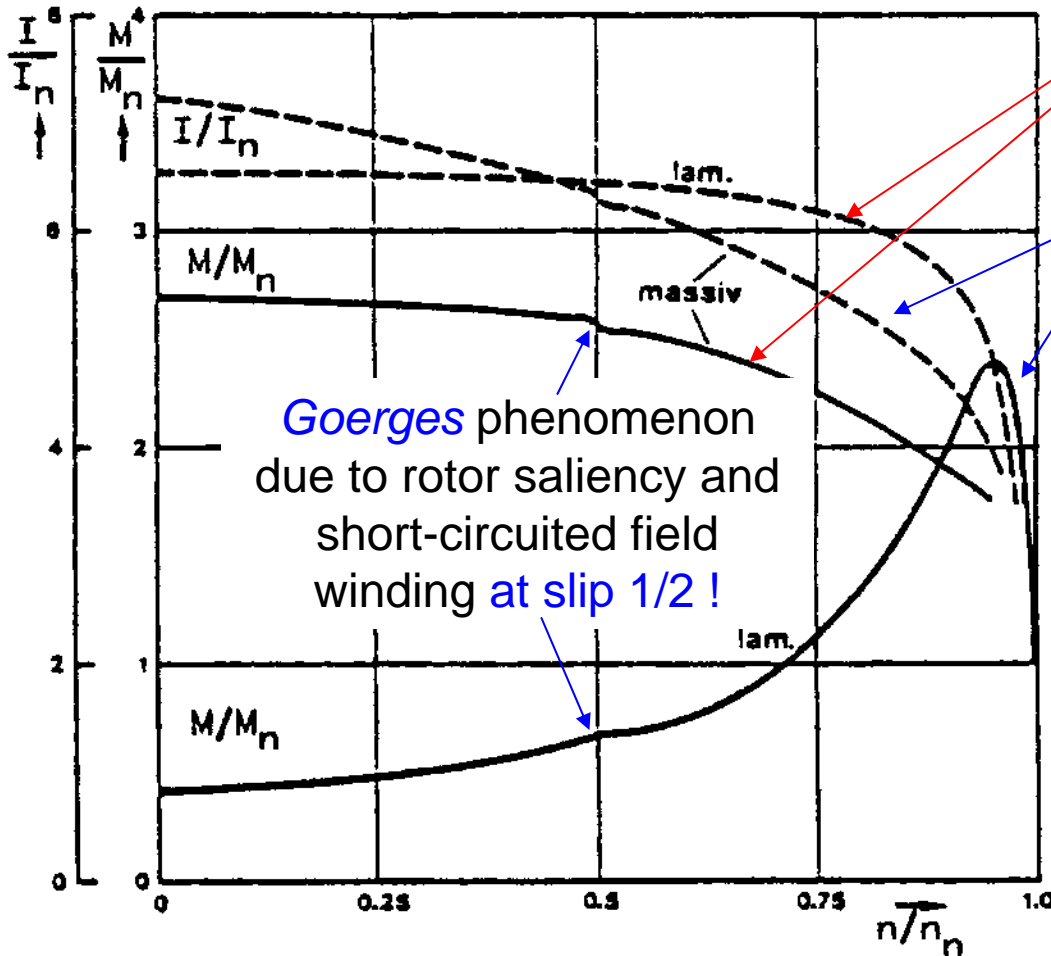


Calculated temperature distribution in deep bar

- At upper bar edge much bigger heating due to current displacement
- Big machines need long time to run up (5 ... 20 s), so equalizing of temperature distribution due to heat conduction in bar
- **Big bar temperature** causes thermal expansion of hot copper:
  - a) bar may be quenched out of slot
  - b) bar-ring welding may tear off !
- **Cages are heat sensitive !**

# 3. Eddy current losses in winding systems

## Asynchronous start-up of synchronous motors



Laminated rotor poles need starting copper cage (= heat sensitive)

- Massive rotor poles do not need rotor cage: massive pole surface (conductive iron) is carrying eddy currents

### Advantages:

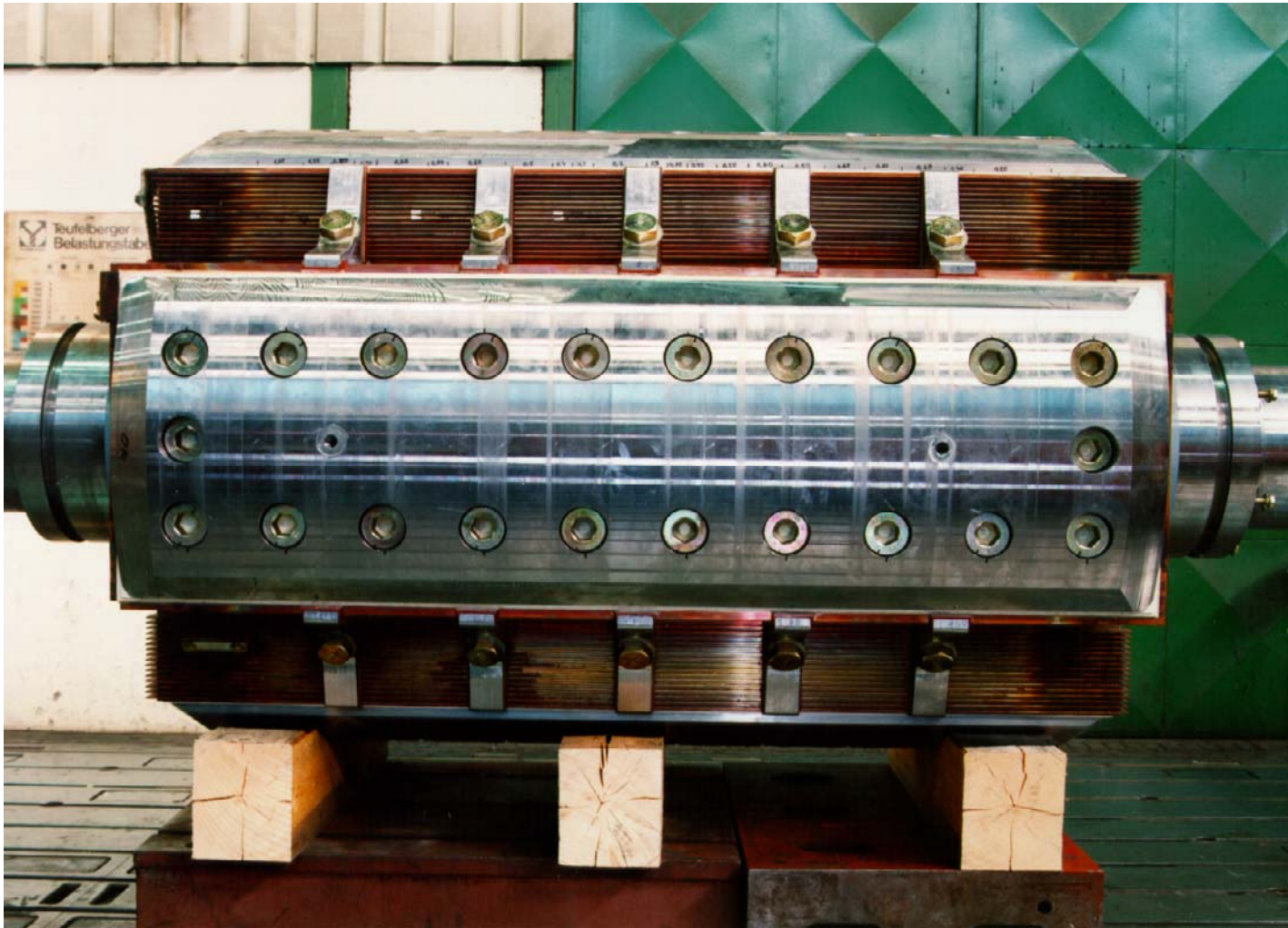
a) no heat expansion problem of cage !

b) 10 ... 20 times bigger iron rotor resistance shifts break down slip to nearly unity = much bigger starting torque.



# 3. Eddy current losses in winding systems

## Massive pole synchronous motors

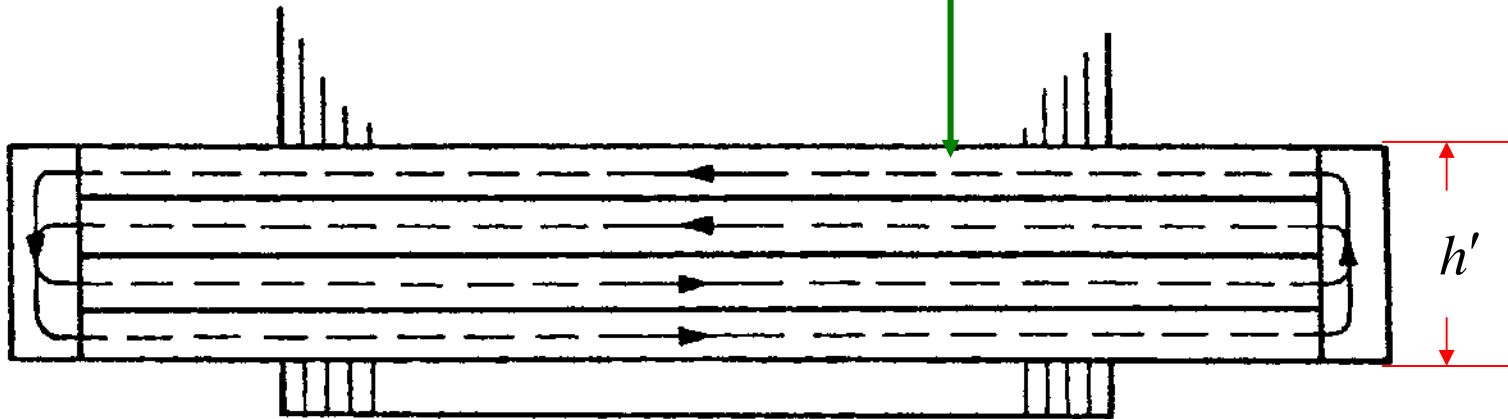


4 pole motor  
Screwed poles

# 3. Eddy current losses in winding systems

## 3.6 Methods to reduce current displacement effect

- Separating massive conductor into several parallel strands **DOES NOT LEAD** to reduction of AC losses, because **circulating eddy currents** will occur via parallel paths = **1<sup>st</sup> order eddy current losses !**



- Eddy currents have to flow also via overhang length, so effective conductivity is decreased by  $l_{Fe}/(l_{Fe} + l_b)$ , but **total conductor height  $h'$  is active !**

$$\xi = h' \cdot \sqrt{\pi f \mu \kappa \cdot \frac{l_{Fe}}{l_{Fe} + l_b} \cdot \frac{b}{b_Q}}$$

- **Facit:** Twisting is also needed !

# 3. Eddy current losses in winding systems

## Transposition of strands in two slots

- Slot stray flux linkage in both coils opposite, so

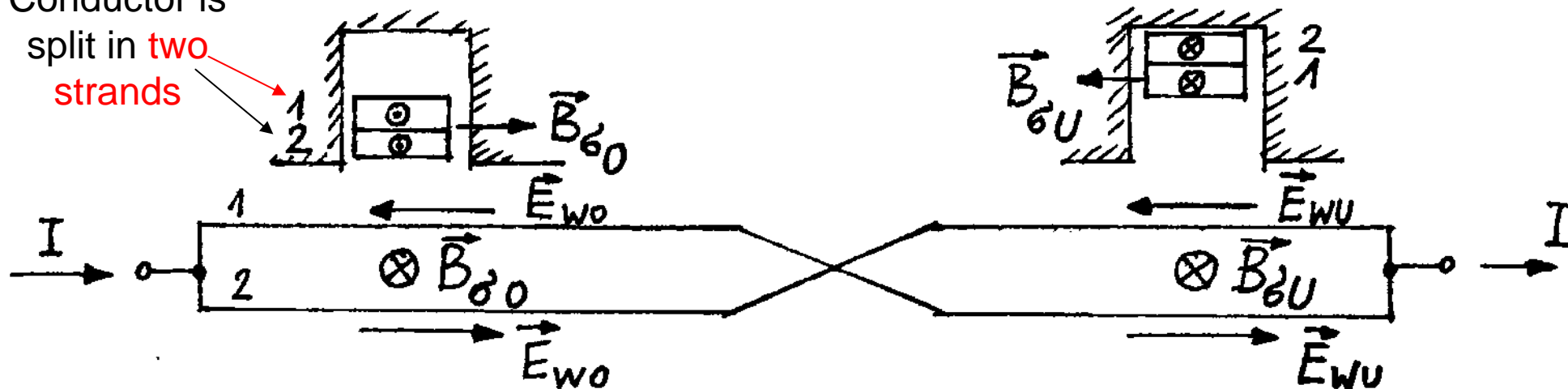
a) in **single layer winding** cancelling of stray flux = **NO** 1<sup>st</sup> order eddy current losses !

$B_{\sigma 0} = B_{\sigma U}$  : induced electric field  $E_{w0} = - E_{wU}$  : total  $E_w = E_{w0} + E_{wU} = 0$  !

b) in **two layer winding** stray flux in upper and lower layer different, so only reduction of total stray flux linkage ( $B_{\sigma 0} > B_{\sigma U}$ ) = reduction, but not full elimination of 1<sup>st</sup> order eddy current losses !

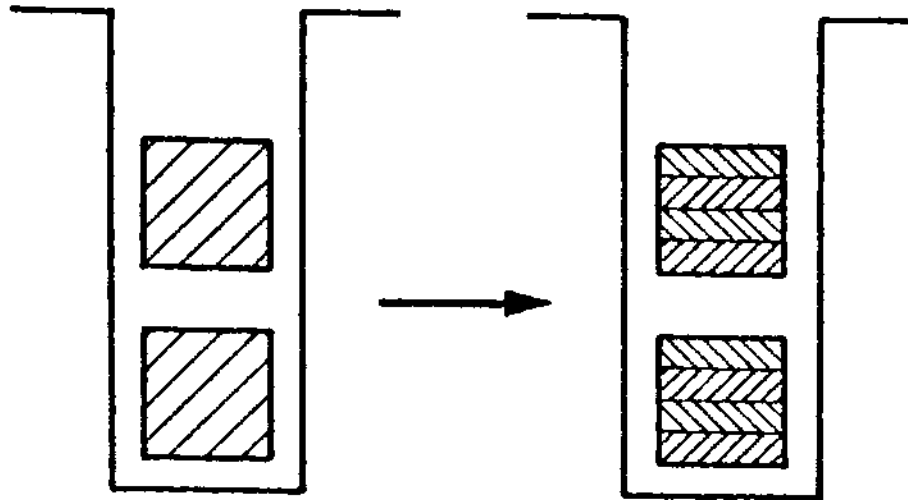
Conductor is

split in **two strands**



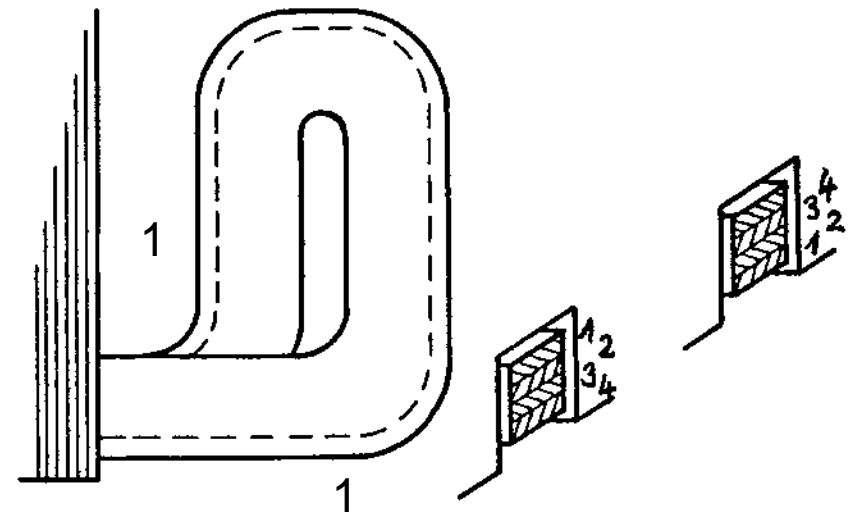
# 3. Eddy current losses in winding systems

## Transposition of strands in form wound coils



Two-layer  
massive bars

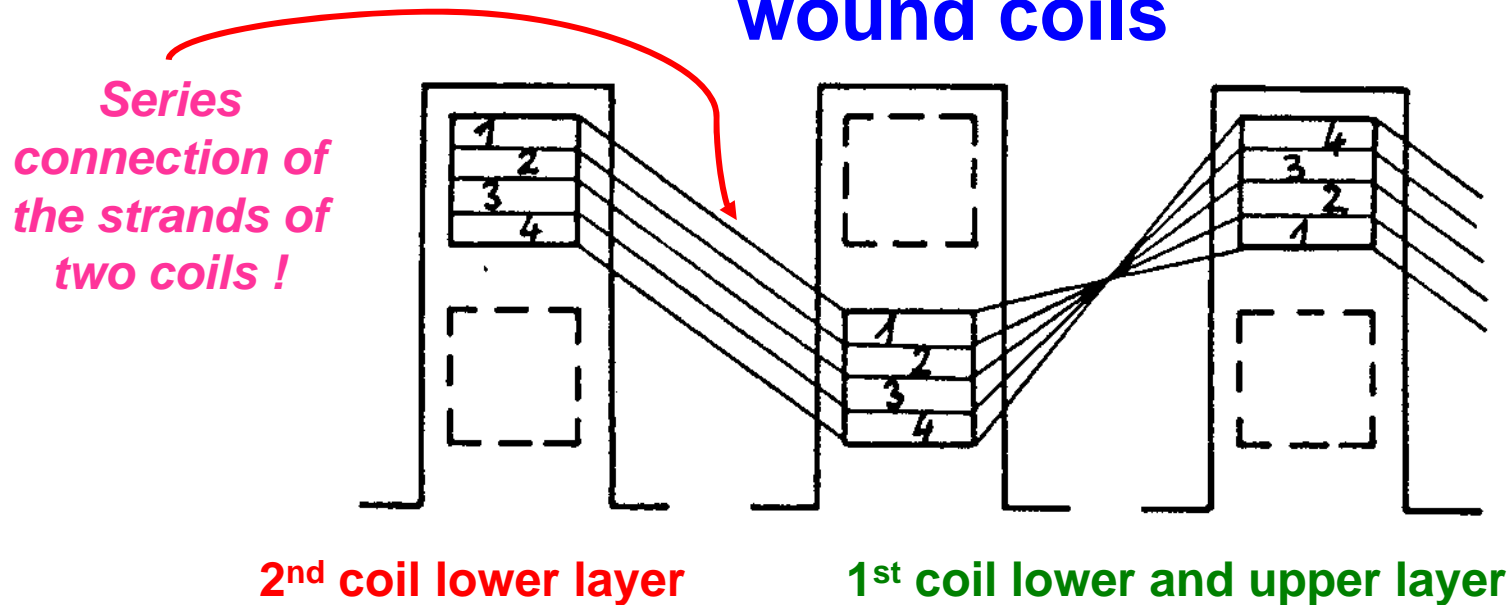
Two-layer  
4 strands per bar



Two-layer form wound coil automatically changes position of strands in slots, so that reduction of stray flux is achieved !

### 3. Eddy current losses in winding systems

## Transposition of strands in series-connected form wound coils



- By connecting only the strands of series-connected two coils without paralleling the strands a complete elimination of slot stray flux is possible also for two-layer winding.
- But this is time-consuming, expensive manufacturing !



# 3. Eddy current losses in winding systems

## 2<sup>nd</sup> order eddy current losses

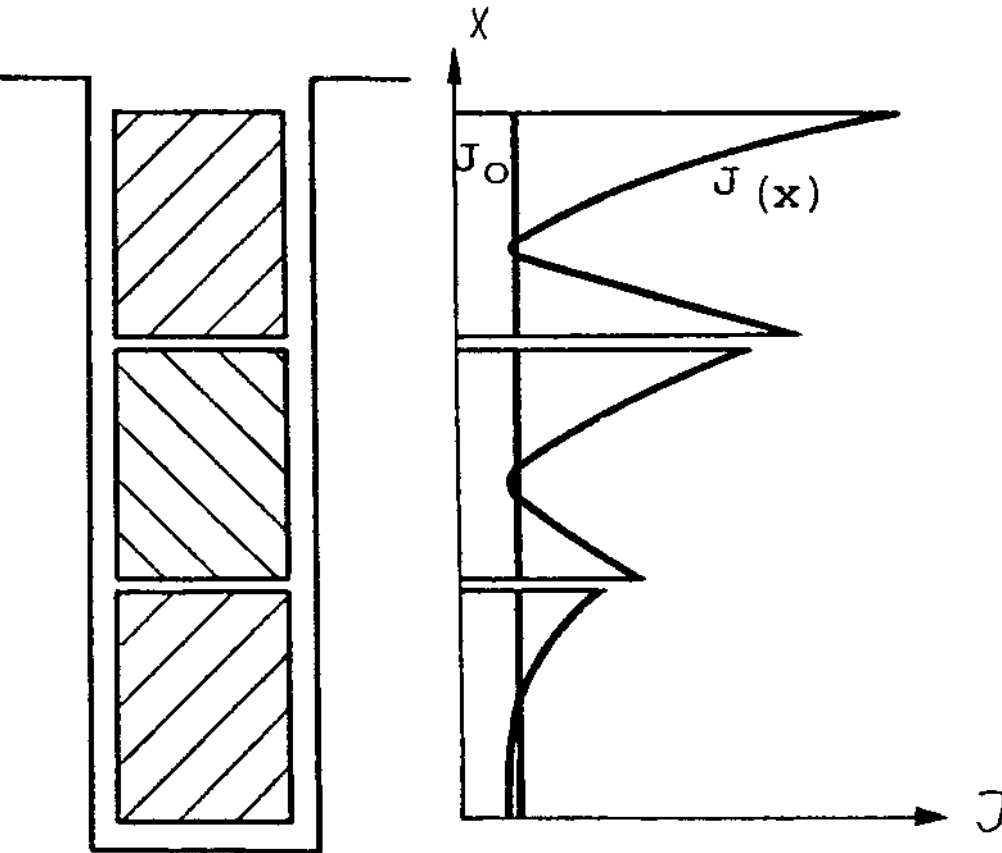
### Coils with $N_c > 1$ number of turns:

- Each turn is a massive conductor
- The lowest conductor experiences only its own slot stray flux:  $k_{R,1} = \phi(\xi)$
- The next conductor experiences its own stray flux and the stray flux, excited by the turn below !

$$k_{R,2} = \phi(\xi) + 2\psi(\xi)$$

- So its eddy current losses are increased !
- $p^{\text{th}}$  turn:  $k_{R,p} = \phi(\xi) + p(p - 1) \cdot \psi(\xi)$

Eddy current losses in massive conductors are called **2<sup>nd</sup> order losses** !



# 3. Eddy current losses in winding systems

## Field's equation for 2<sup>nd</sup> order eddy current losses

- Increase of losses in  $p^{\text{th}}$  turn due to eddy currents:

$$k_p = \varphi(\xi) + p(p-1)\psi(\xi)$$

$$\varphi(\xi) = \xi \cdot \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}$$

$$\psi(\xi) = 2\xi \frac{sh\xi - \sin \xi}{ch\xi + \cos \xi}$$

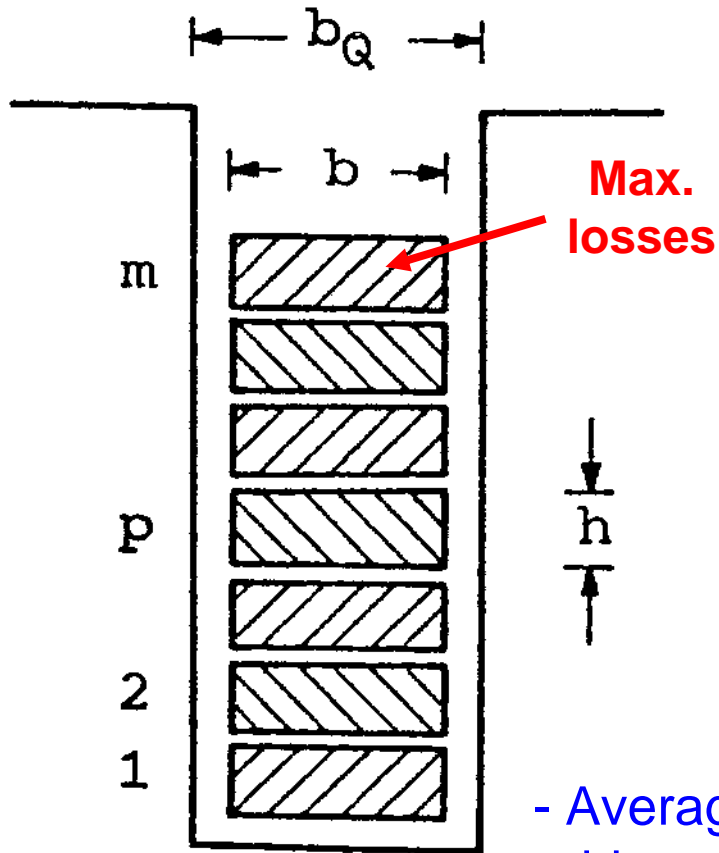
$$\xi = \frac{h}{d_E} = h \sqrt{\pi f \mu \kappa \frac{b}{b_Q}}$$

- Average increase of losses per turn ( $m$  turns assumed):

$$k_m = \frac{1}{m} \sum_{p=1}^m k_p = \varphi(\xi) + \frac{m^2 - 1}{3} \psi(\xi)$$

- Average of slot (eddy currents) and winding overhangs (no eddy currents):

$$k_R = \frac{k_m \cdot l_{Fe} + l_b}{l_{Fe} + l_b}$$

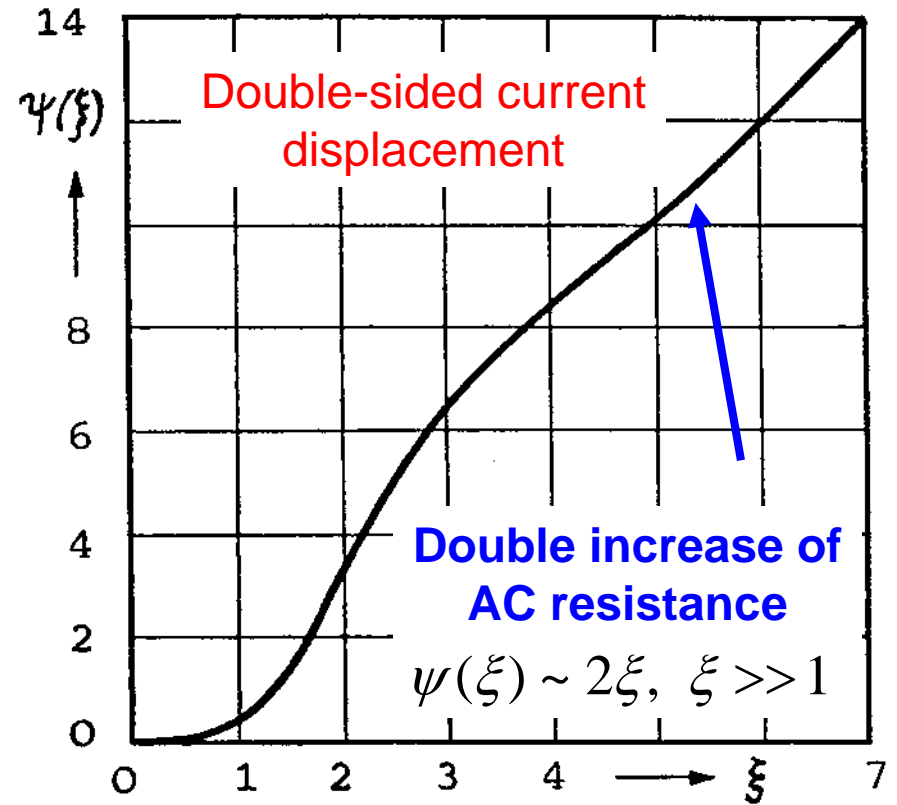
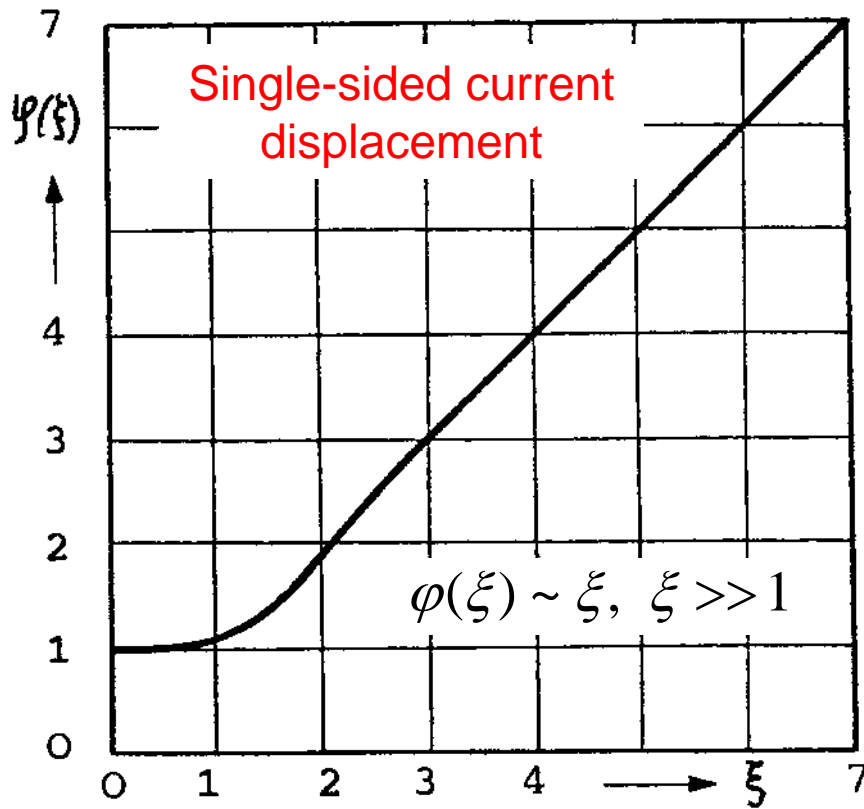


# 3. Eddy current losses in winding systems

## Field's loss functions

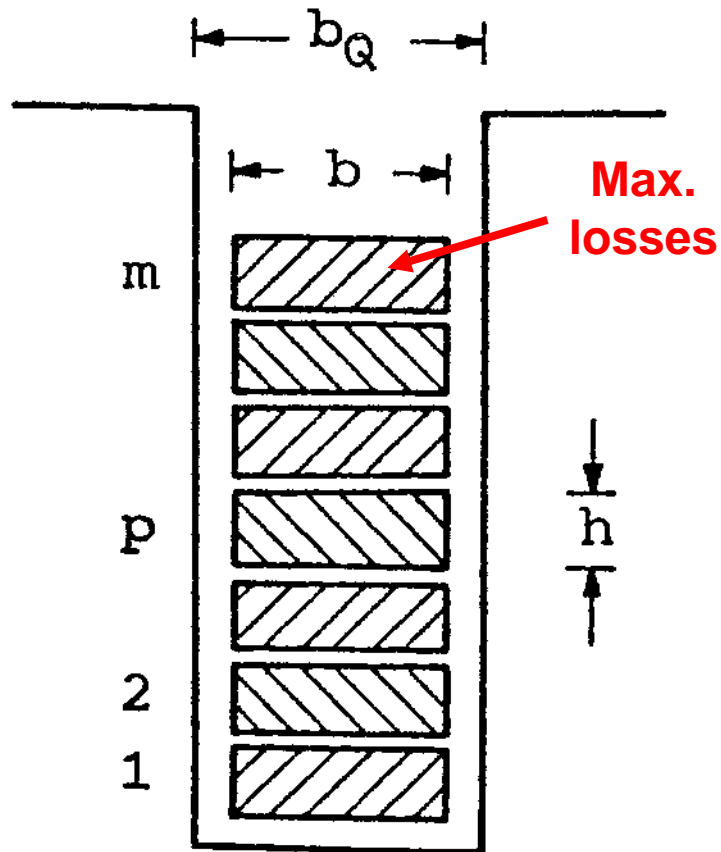
$$\varphi(\xi) = \xi \cdot \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}$$

$$\psi(\xi) = 2\xi \frac{\operatorname{sh} \xi - \sin \xi}{\operatorname{ch} \xi + \cos \xi}$$



# 3. Eddy current losses in winding systems

## Field's equation for 2<sup>nd</sup> order eddy current losses



### - Number $m$ :

Single layer winding:  $m = N_c$  is number of turns per coil

Two-layer winding:  $m = 2N_c$  is TWICE number of turns per coil

- Loss increase in top conductor  $p = m$ :

$$k_{p=m} = \varphi(\xi) + m(m-1)\psi(\xi)$$

Top conductor suffers maximum additional losses due to eddy currents !

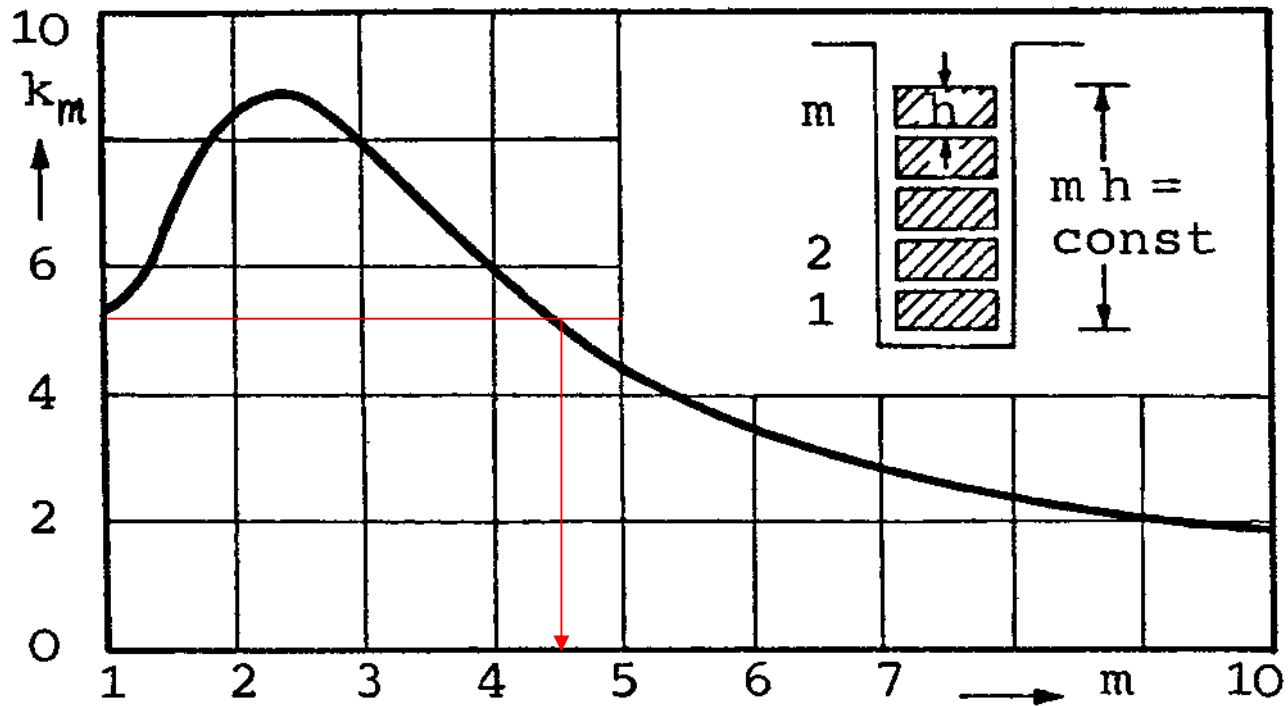
# 3. Eddy current losses in winding systems

## 2<sup>nd</sup> order losses with increased number of turns $m$

Example:  $b/b_Q = 0.8$ , 50 Hz

Total slot height  $m \cdot h = 60 \text{ mm} = \text{constant}$ , but  $m$  increases !

Is  $k_m$  increasing / decreasing, as conductor height  $h = 60\text{mm}/m$  decreases ?



- Eddy current losses increase from one conductor to two conductors per slot !

- The average losses per conductor  $k_m/m$  decrease at constant current density  $J$  !

# 3. Eddy current losses in winding systems

## Reduction of 2<sup>nd</sup> order losses (1)

- Increase number of turns per slot to reduce conductor height  $h$ :

$$\text{Aim: } \xi = \frac{h}{d_E} = h \sqrt{\pi f \mu \kappa \frac{b}{b_Q}} < 0.3$$

- Example: 12-pole synchronous generator:  $n = 500/\text{min}$ ,  $2p = 12$ ,  $f = 50 \text{ Hz}$ 
  - Stator winding:  $N_c = 2$ ,  $q = 2$ ,  $W = 5/6 \tau_p$ ,  $a = 2$ ,  $\tau_p = 0.5 \text{ m}$ ,  $l = 1 \text{ m}$ ,  $m = 2N_c = 2 \times 2 = 4$
  - Number of turns per phase:  $N = 2pqN_c / a = 12 \cdot 2 \cdot 2 / 2 = \underline{\underline{24}}$

If number of parallel winding branches per phase  $a$  is increased,

- the number of turns per slot  $N_c$  increases,
- the current per winding branch  $I_c = I_s / a$  decreases and so does  $h$ .
- $a = 6$  instead of 2:  $m = 2N_c = 12$  is tripled,  $h$  is one third of old value

$$N = 2pqN_c / a = 12 \cdot 2 \cdot 6 / 6 = \underline{\underline{24}}$$

**Result: 2<sup>nd</sup> order additional losses decreases to about 30% !**



# 3. Eddy current losses in winding systems

## Reduction of 2<sup>nd</sup> order losses (2)

Example: 12-pole synchronous generator:  $n = 500/\text{min}$ ,  $2p = 12$ ,  $f = 50 \text{ Hz}$

- Stator winding:

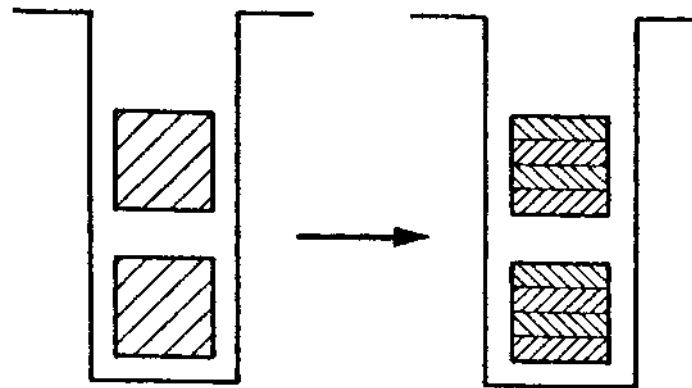
a)  $N_c = 1$ ,  $q = 2$ ,  $W = 5/6\tau_p$ ,  $a = 1$ ,  $\tau_p = 0.5 \text{ m}$ ,  $l = 1 \text{ m}$ ,  $m = 2N_c = 2 \times 1 = 2$

- Number of turns per phase:  $N = 2pqN_c / a = 12 \cdot 2 \cdot 1 / 1 = \underline{\underline{24}}$

b)  $N_c = 4$ ,  $q = 2$ ,  $W = 5/6\tau_p$ ,  $a = 4$ ,  $\tau_p = 0.5 \text{ m}$ ,  $l = 1 \text{ m}$ ,  $m = 2N_c = 2 \times 4 = 8$

- Number of turns per phase:  $N = 2pqN_c / a = 12 \cdot 2 \cdot 4 / 4 = \underline{\underline{24}}$

**Result: 2<sup>nd</sup> order additional losses decreases to about 25% !**



# 3. Eddy current losses in winding systems

## Critical conductor height for coils $N_c > 1$ (1)

- For  $\xi < 1$ : *Taylor*-series:  $\varphi(\xi) \approx 1 + \frac{4}{45} \xi^4$      $\psi(\xi) \approx \frac{\xi^4}{3}$

$$k_m \approx 1 + \frac{4}{45} \cdot \xi^4 + \frac{m^2 - 1}{3} \cdot \frac{\xi^4}{3} = 1 + \frac{m^2 - 0.2}{9} \cdot \xi^4$$

- Constant current per conductor  $I$ , constant number of conductors per slot  $m$ :

Total losses (with  $h = \xi \cdot d_E$ ):  $P_1 = k_m \cdot P_0 = \frac{I^2 L}{b \cdot h \cdot \kappa} \cdot k_m \approx \frac{I^2 L}{b \cdot d_E \cdot \kappa} \cdot \left( \frac{1}{\xi} + \frac{m^2 - 0.2}{9} \cdot \xi^3 \right)$

- **Loss minimum** at  $dP_1/d\xi = 0$ :  $\xi_{crit} = \frac{h_{crit}}{d_E} \approx 4 \sqrt{\frac{3}{m^2 - 0.2}} \approx 4 \sqrt{\frac{3}{m^2}} = \frac{1.32}{\sqrt{m}}$

- **Optimum increase of 2<sup>nd</sup> order current displacement losses:**

$$k_{m,opt} = k_m(\xi_{crit}) \approx 1 + \frac{m^2 - 0.2}{9} \cdot \xi_{crit}^4 \approx 1 + \frac{m^2 - 0.2}{9} \cdot \frac{3}{m^2 - 0.2} = 1 + \frac{1}{3} = \underline{\underline{1.33}}$$





# 3. Eddy current losses in winding systems

## Critical conductor height for coils $N_c > 1$ (2)

- Critical conductor height  $h_{crit}$  depends on number of conductors  $m$  per slot one above the other:

$$\xi_{crit} = \frac{h_{crit}}{d_E} = \frac{1.32}{\sqrt{m}}$$

- Height  $h_{crit}$  is smaller, the bigger  $m$  is !

**Critical total slot height  $H_{krit}$ :**  $H_{crit} = m \cdot h_{crit} \approx m \cdot d_E \cdot \frac{1.32}{\sqrt{m}} = 1.32 \cdot \sqrt{m} \cdot d_E$

### Facit:

As total critical slot height rises with number  $m$ , the total amount of copper height may be increased !

Applicable for “small” machines with high voltage  $U$ :

“Small” machine: Flux per pole  $\Phi$  is small: So need for high  $N_s$  and  $N_c$  !

High  $N_c =$  high  $m$  !

$$U \cong \omega \cdot k_w N_s \cdot \Phi$$



# 3. Eddy current losses in winding systems

## Bar windings : $N_c = 1$

- **Big generators:** Flux per pole  $\Phi$  is big, so  $N_s$  is low:  $U \cong \omega \cdot k_w N_s \cdot \Phi$

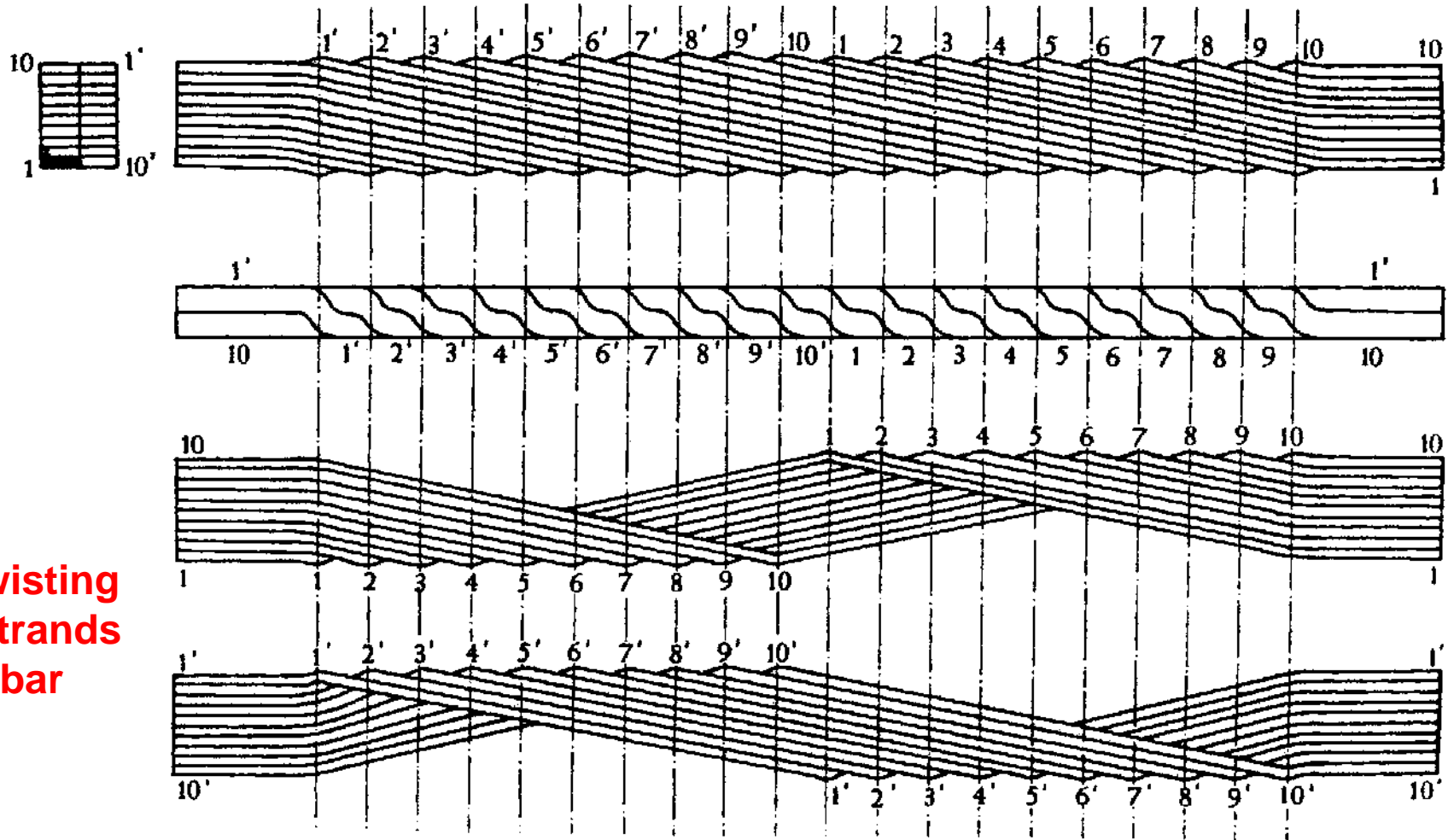
Often  $N_c = 1$ , especially with two-pole turbine generators, as maximum number of parallel paths is  $a = 2$ .

- Partition of bar into small parallel connected conductors leads to big 1<sup>st</sup> order eddy currents, so **twisting of these parallel conductors** is necessary to eliminate slot stray flux linkage.
- Patent of *Ludwig Roebel* (German engineer, at *BBC company*): Twisting of profile copper conductors in rectangular slot **eliminates 1<sup>st</sup> order eddy currents**.
- In *Roebel-bars* only 2<sup>nd</sup> order eddy currents remain !



# 3. Eddy current losses in winding systems

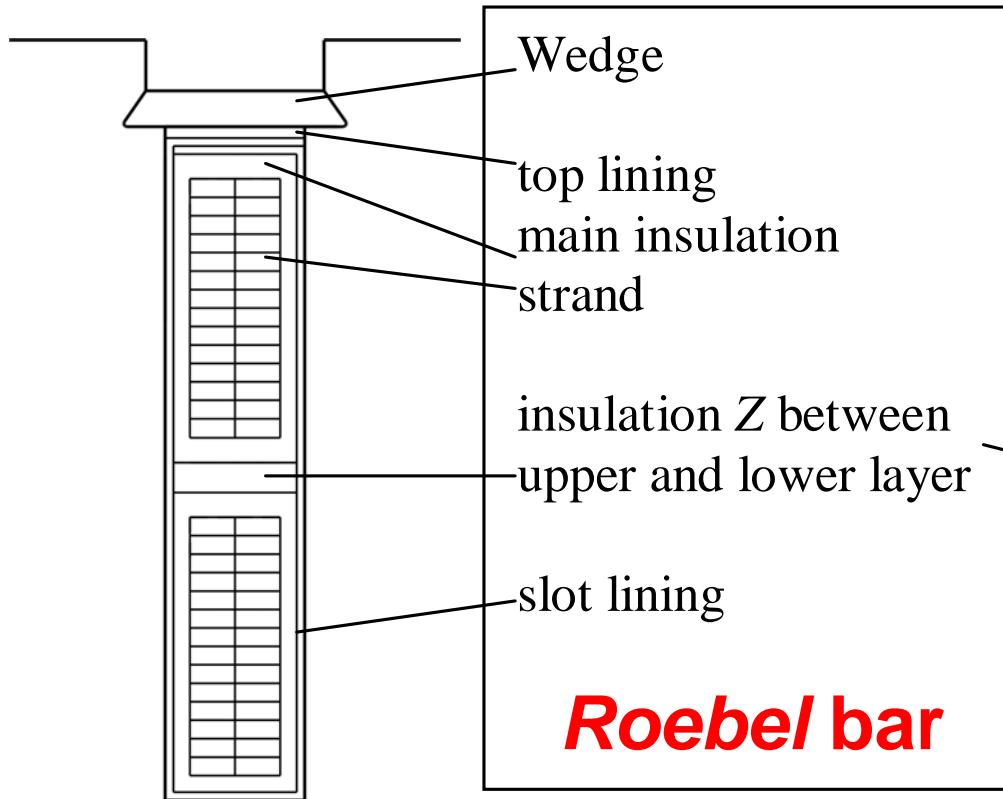
Roebel bar = 20 parallel twisted conductors (strands)



360° twisting  
of 20 strands  
per bar

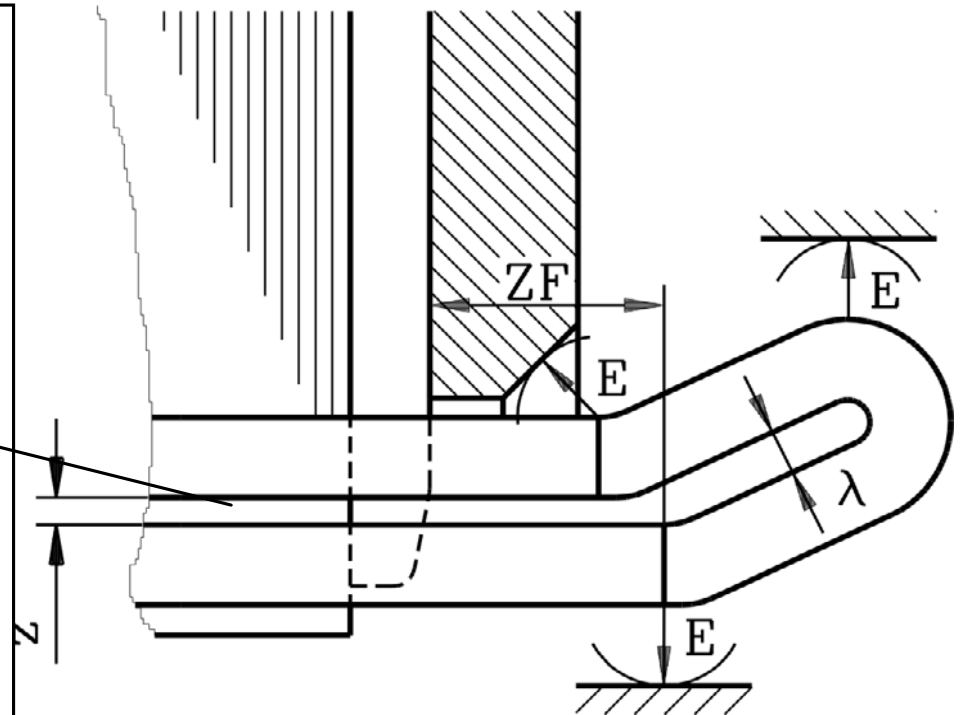
# 3. Eddy current losses in winding systems

## Roebel bar cross section and winding overhang



Cross section in slot with  
24 strands per bar

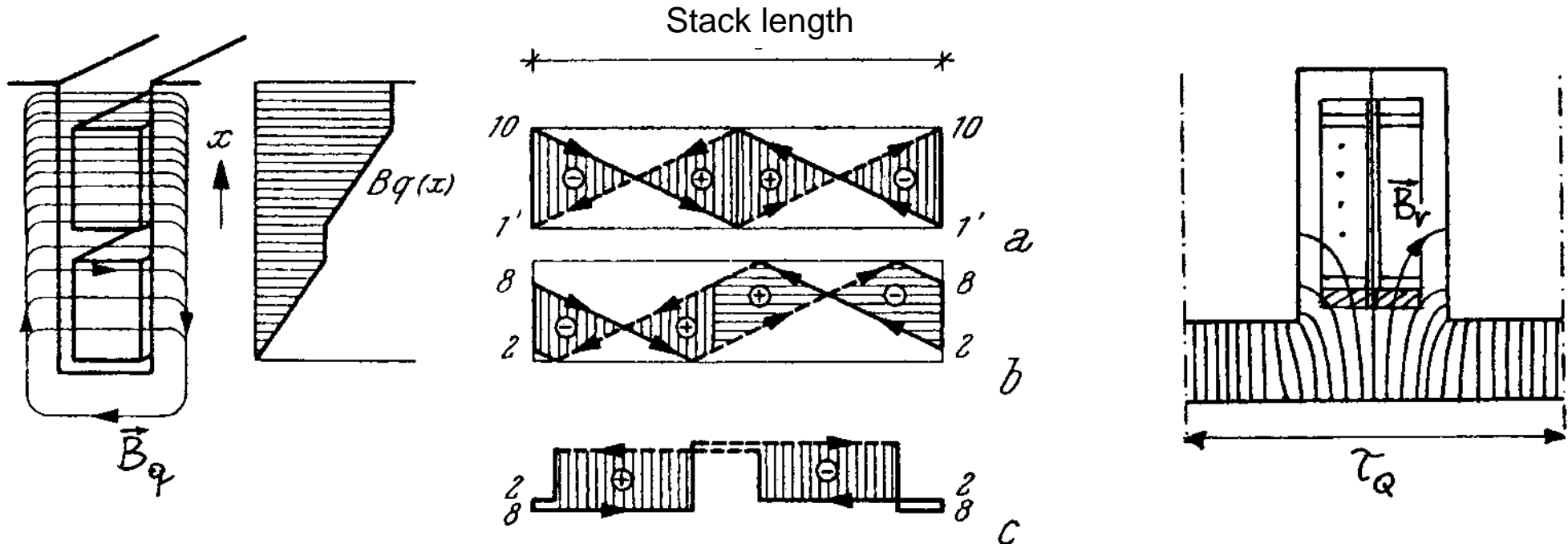
$$N_c = 1$$



Winding overhang with clearing  $E$  to end shields (on earth potential) and clearing  $ZF$  of end of main insulation from pressing construction of iron stack end

# 3. Eddy current losses in winding systems

## Slot stray flux linkage elimination in *Roebel* bars



- a), b) Each arbitrarily chosen pair of parallel strands (e.g. 1 and 10, 2 and 8, ...) forms loops, where **positive and negative oriented areas have the same shape and relative position!** So the linkage of linear increasing slot flux density  $B_q$  is cancelled completely for each pair!
- c) The radial slot flux  $B_r$  of main field experiences likewise for each arbitrarily chosen pair of parallel strands (e.g. 1 and 10, 2 and 8, ...) loops with **positive and negative oriented equal areas:** So also radial flux linkage is cancelled completely for each pair of strands!

# 3. Eddy current losses in winding systems

## Example:

ROEBEL-bars: 2x12 strands: strand geometry  $h_T = 2$  mm,  $f = 60$  Hz,  $\vartheta_{Cu} = 100$  °C,  $b/b_Q = 0.7$ , winding overhang length ratio:  $l_b/l_{Fe} = 0.65$ .

$$\xi = \frac{h_T}{d_E} = h_T \sqrt{\pi f \mu_0 \kappa \frac{b}{b_Q}} = 0.002 \sqrt{\pi \cdot 60 \cdot 4\pi 10^{-7} \cdot 43.4 \cdot 10^6 \cdot 0.7} = 0.17$$

$m = 24$  strands one above the other ! 2<sup>nd</sup> order eddy current losses:

$$\text{FIELD's formulas: } \varphi(\xi) \approx 1 + \frac{4}{45} \xi^4 = 1.000074, \quad \psi(\xi) \approx \frac{\xi^4}{3} = 0.000276$$

$$k_m = \varphi(\xi) + \frac{m^2 - 1}{3} \psi(\xi) = 1.000074 + \frac{24^2 - 1}{3} \cdot 0.000276 = 1.000074 + 0.053 = \underline{\underline{1.0531}}$$

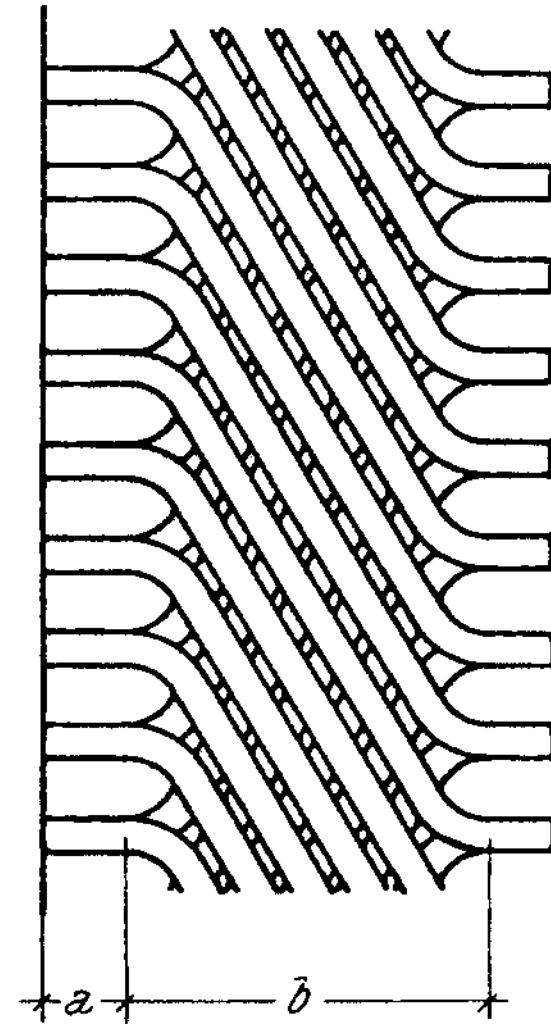
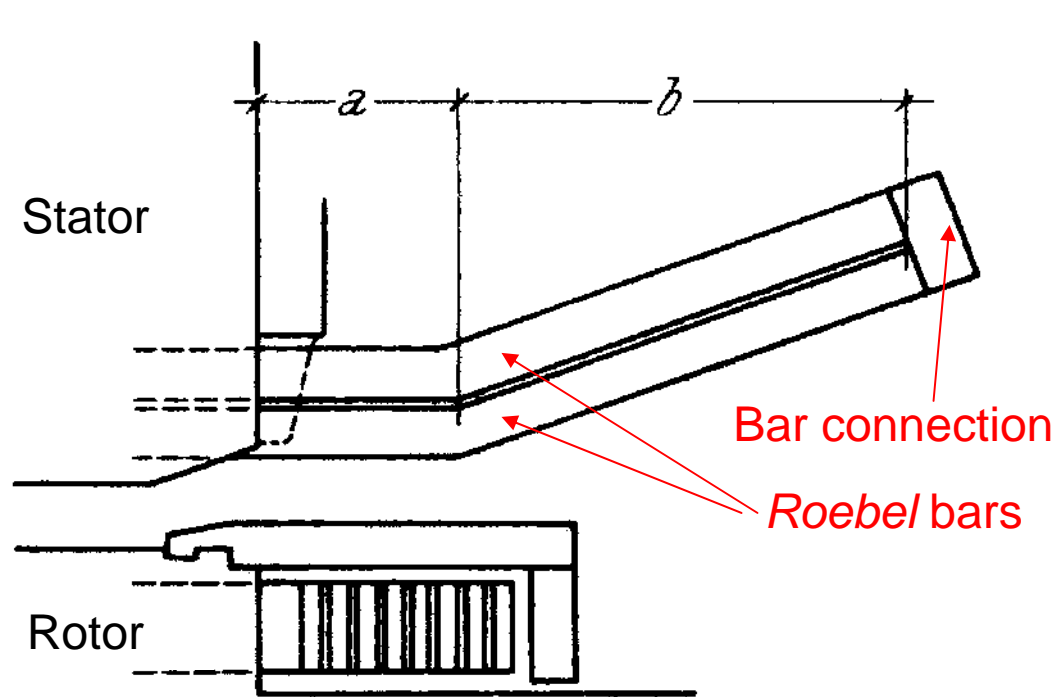
$$\text{Average value per turn: } k_R = \frac{k_m + (l_b / l_{Fe})}{1 + (l_b / l_{Fe})} = \frac{1.0531 + 0.65}{1.65} = \underline{\underline{1.032}}$$

**Average increase of losses is only 3.2%. No 1<sup>st</sup> order eddy current occur !**



# 3. Eddy current losses in winding systems

## Winding overhang geometry with *Roebel* bars



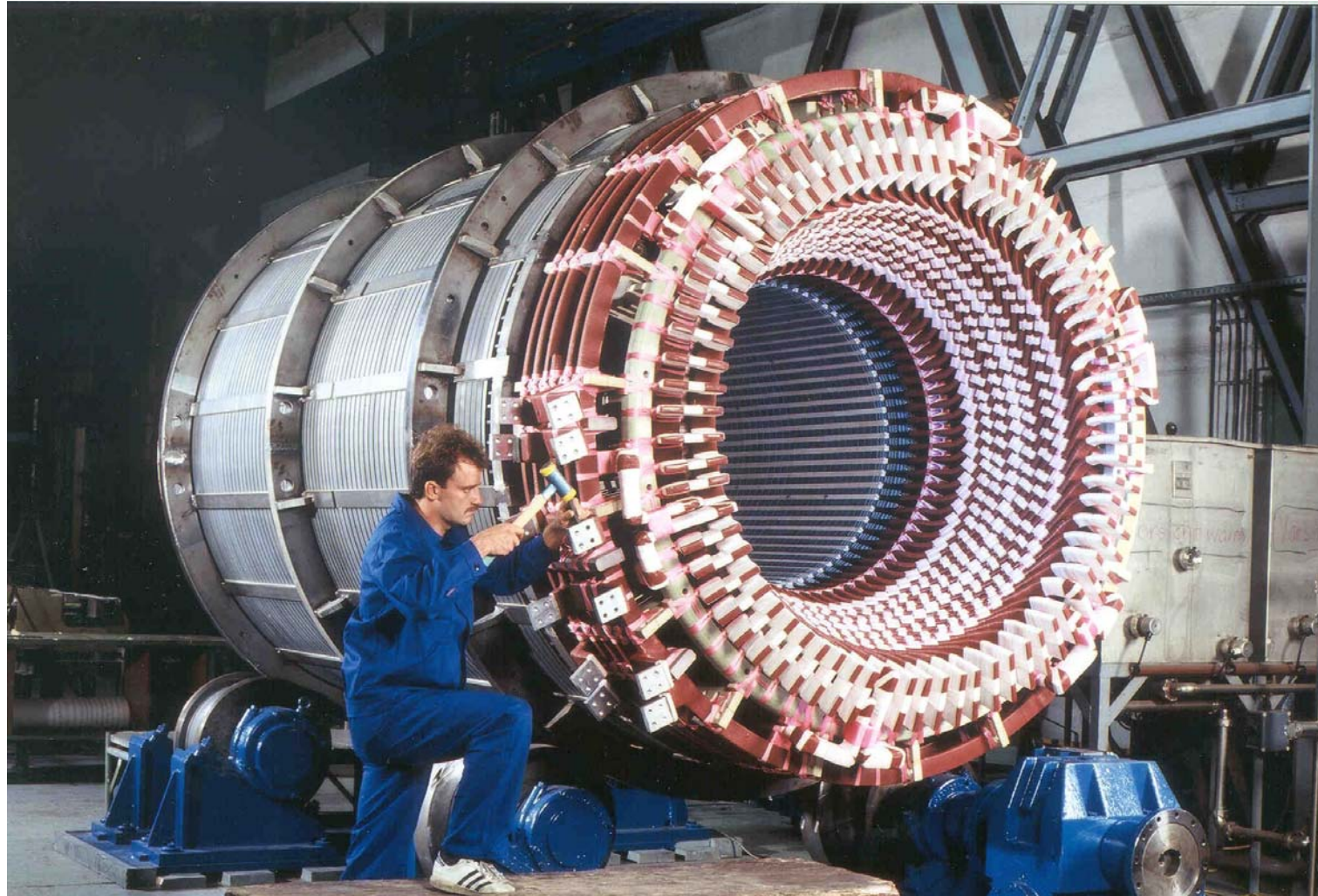
### Winding overhang geometry of a two-layer winding of a turbine generator with *Roebel* bars

- (a): straight part of overhang
- (b): curved part of overhang

# 3. Eddy current losses in winding systems

## Winding overhang geometry with *Roebel* bars

Air-cooled 2-pole turbine generator

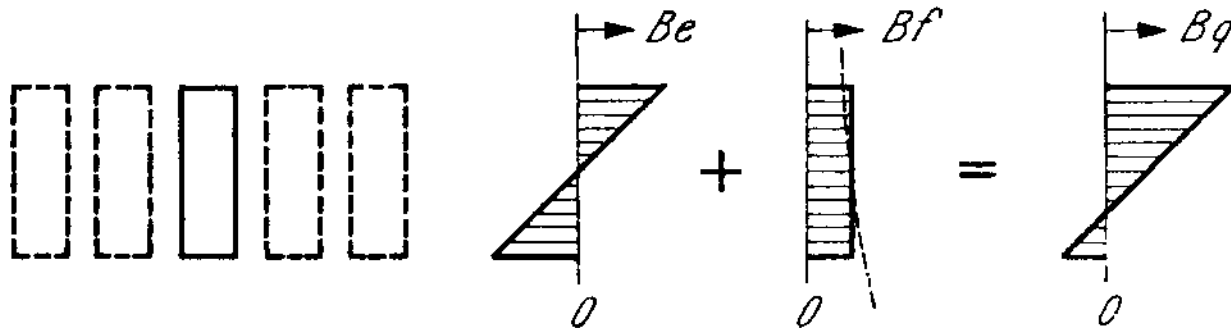




# 3. Eddy current losses in winding systems

## Winding overhang stray flux in *Roebel* bars

One layer of bars in section (a)



- Winding overhang stray field much smaller than slot stray field, so twisting of winding overhang in hydro generators not done !

Note: Low speed = high pole count = small pole pitch = short winding overhang: eddy current losses in winding overhang of minor importance ! But in two-pole turbine generators: LONG winding overhangs, so transposition of strands is NECESSARY !

- Cross section of bars e.g. in section (a): Transversal component of stray field  $B_q$ :

$B_e$ : Self field of on bar in bar axis

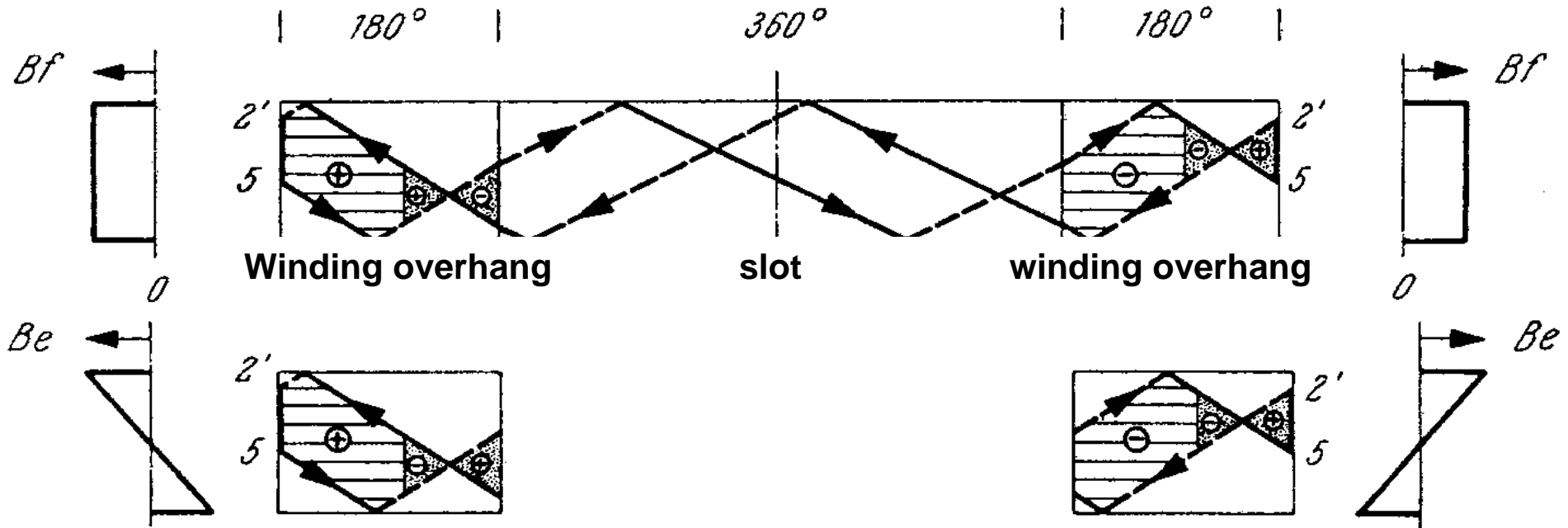
$B_f$ : External field of bars of other layer in bar axis

- Condition for elimination 1st order eddy currents in winding overhangs:

Twisting of strands in winding overhang must eliminate stray flux linkage of resulting transversal stray field  $B_q$  (=sum of  $B_e$  and  $B_f$ ), hence of self field AND of external field !

# 3. Eddy current losses in winding systems

## Winding overhang stray flux linkage elimination in Roebel bars (1)

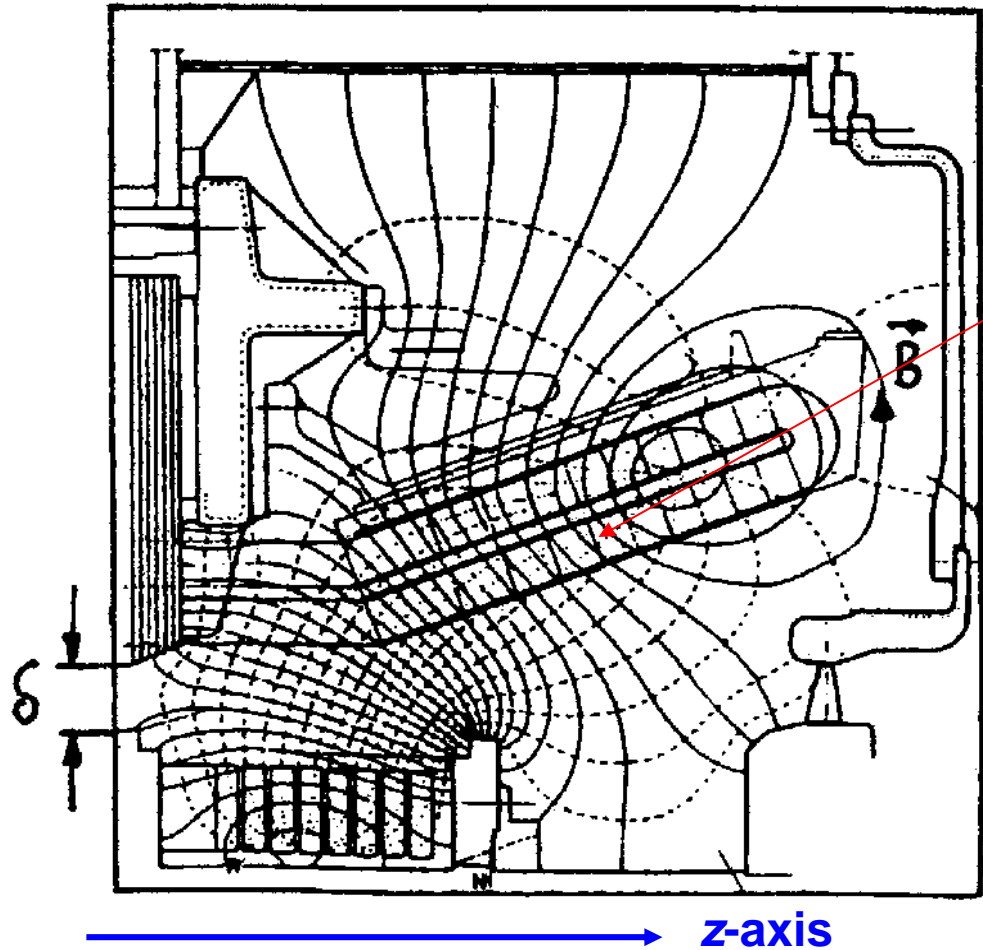


**Example:** 180° twisting of strands in winding overhangs (180°/360°/180°):

- Each arbitrarily chosen pair of parallel strands (e.g. 2 and 5, ...) forms loops, where **positive and negative oriented areas have the same shape and relative position !** So
  - the linkage of linear changing self-field flux density  $B_e$  and
  - the linkage of the (more or less constant) external flux density  $B_f$  is cancelled **completely for each pair !**

# 3. Eddy current losses in winding systems

## Stray field in winding overhang under load



- Winding overhang of stator experiences also strong **radial stray field component  $B_r$** , which has to be taken into account for stray losses in **very big machines**

- In **smaller machines** it may be neglected, as the stray flux linkage area for radial field  $B_r$  **is much smaller** than for transverse field  $B_q$  !

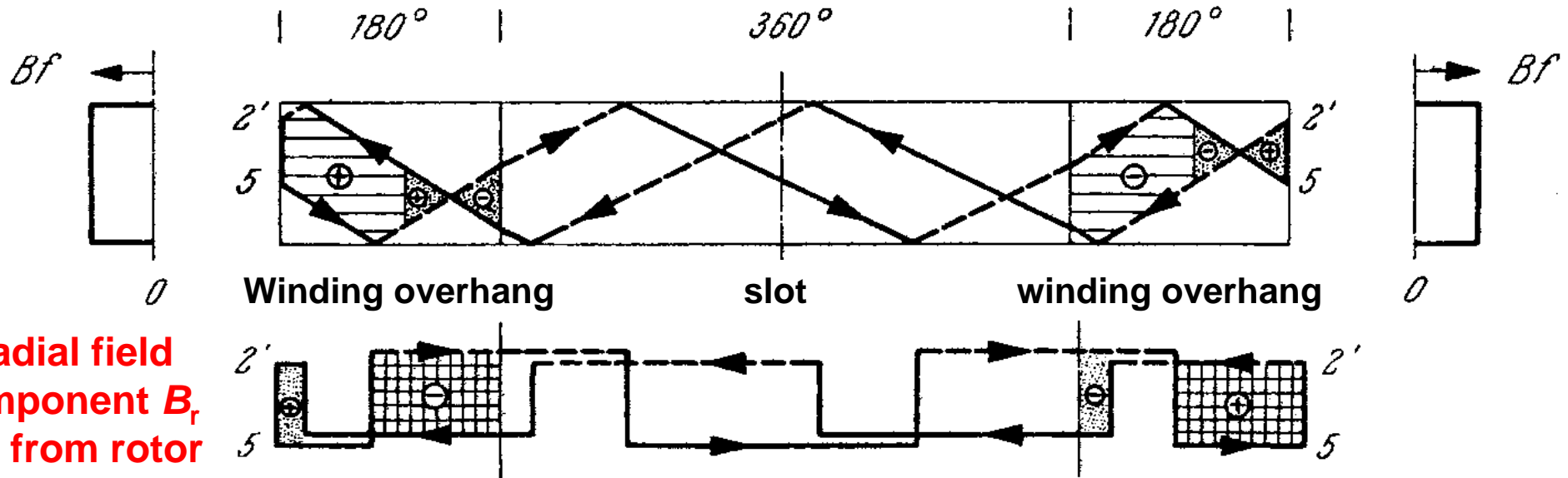
- E.g. section (a): Length  $L_a$ :

$$\text{Area for } B_r : 2 \times b_L \times L_a$$

$$\text{Area for } B_q : (m/2) \times h_L \times L_a$$

# 3. Eddy current losses in winding systems

## Winding overhang stray flux linkage elimination in Roebel bars (2)



Radial field component  $B_r$ , e.g. from rotor

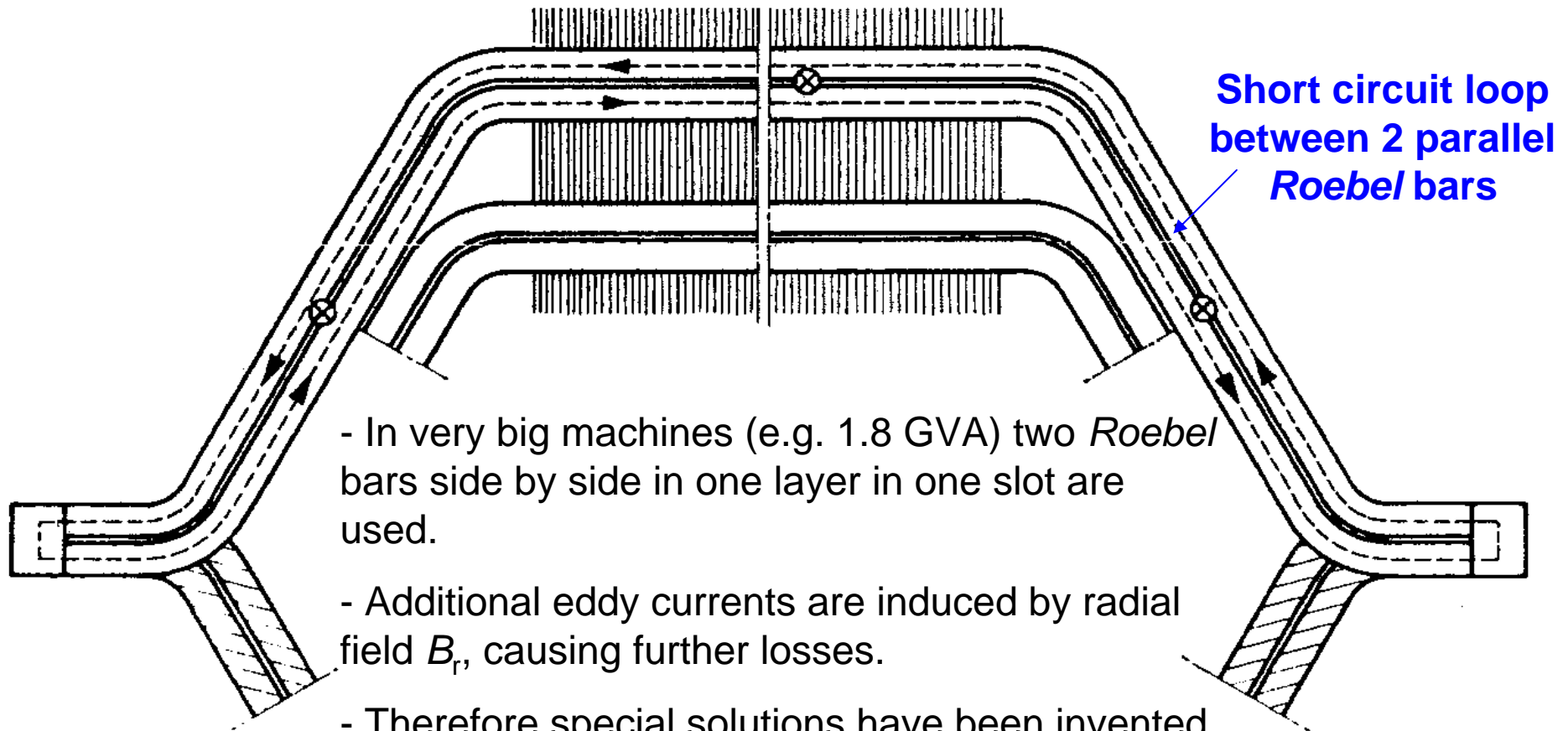
**Example:** 180° twisting of strands in winding overhangs (180°/360°/180°):

- If  $B_r$  is constant along z-axis, then its flux linkage is also cancelled ! Usually  $B_r(z)$  decreases with increasing distance from iron stack ! So 180°/360°/180° does not fully cancel radial stray flux linkage !

**Note:** Special (very expensive) 180°/540°/-180° eliminates also flux linkage due to  $B_r(z)$ , which is varying with co-ordinate z !

### 3. Eddy current losses in winding systems

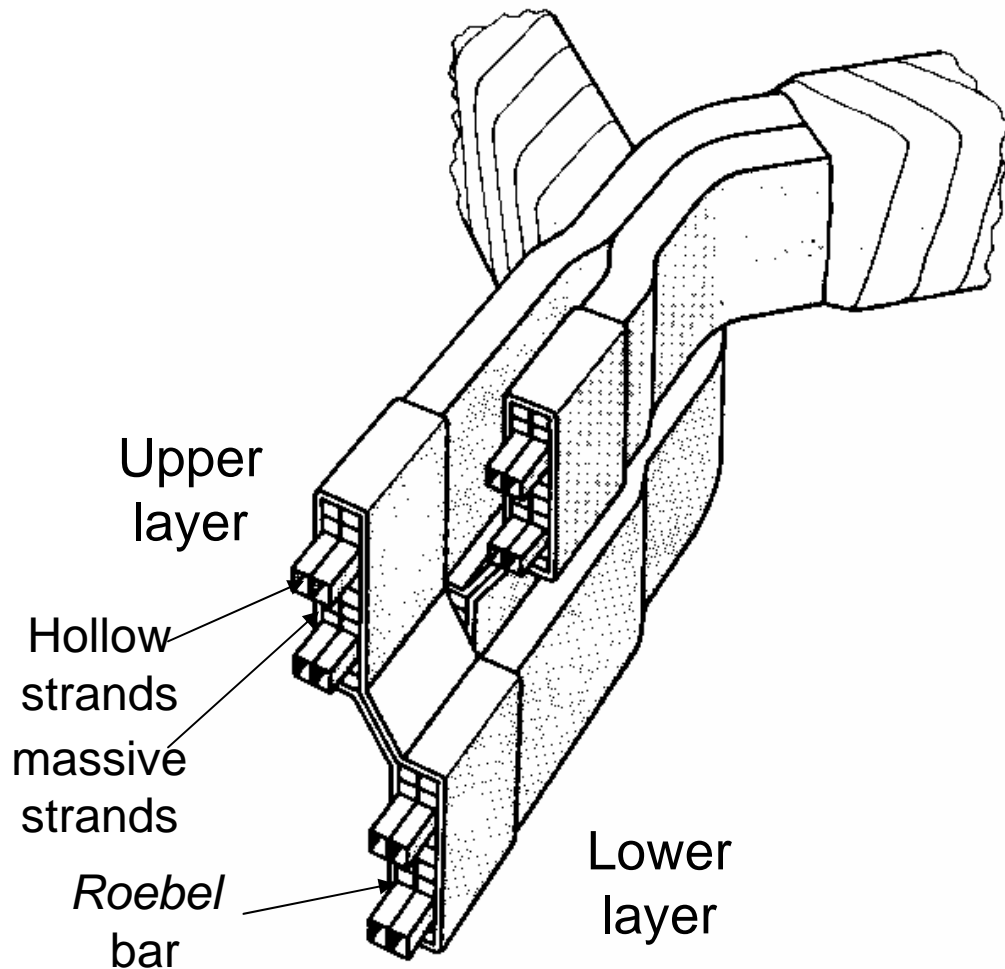
#### Parallel connection of 2 *Roebel* bars per slot



- In very big machines (e.g. 1.8 GVA) two *Roebel* bars side by side in one layer in one slot are used.
- Additional eddy currents are induced by radial field  $B_r$ , causing further losses.
- Therefore special solutions have been invented, e.g. transposed parallel connection of *Roebel* bars !

# 3. Eddy current losses in winding systems

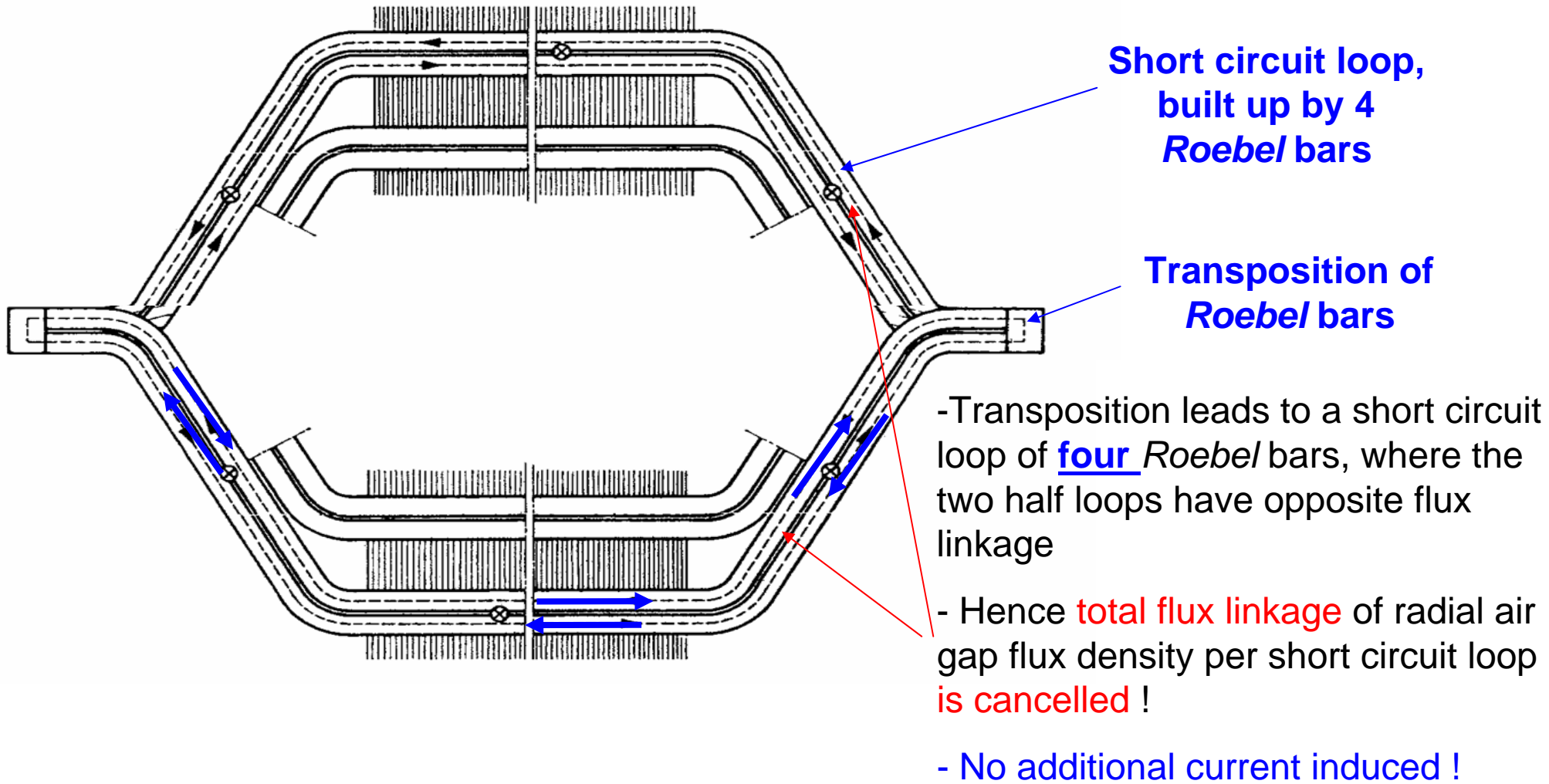
## Crosswise connection of 2 parallel *Roebel* bars per slot



- Transposed parallel connection of 2 parallel *Roebel* bars per slot!
- Transposition is done in the winding overhang!
- Transposition leads to a short circuit loop of **four** *Roebel* bars, where the two half loops have opposite flux linkage
- Hence total flux linkage of radial air gap flux density per short circuit loop is cancelled !
- **No additional current induced !**

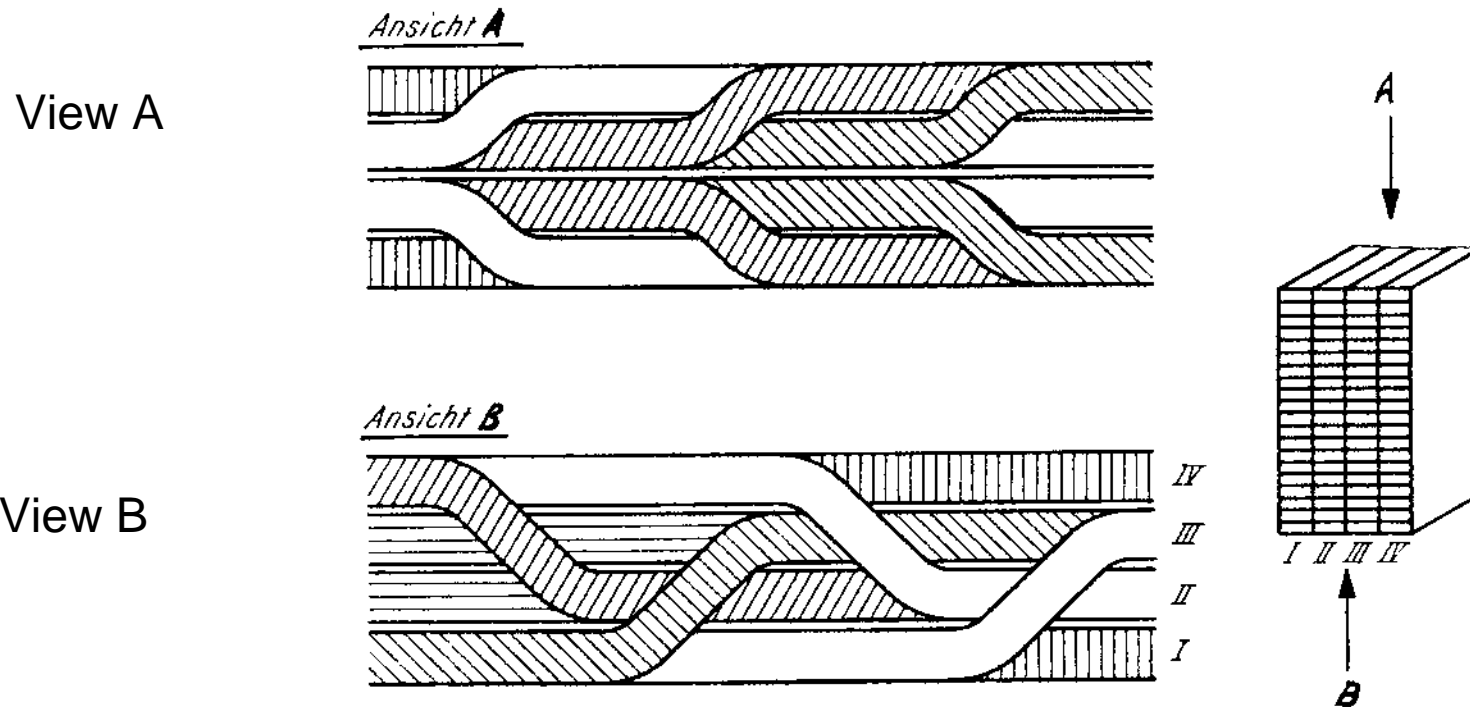
# 3. Eddy current losses in winding systems

## Crosswise connection of 2 parallel *Roebel* bars per slot



# 3. Eddy current losses in winding systems

## Complete twisting of four rows of strands

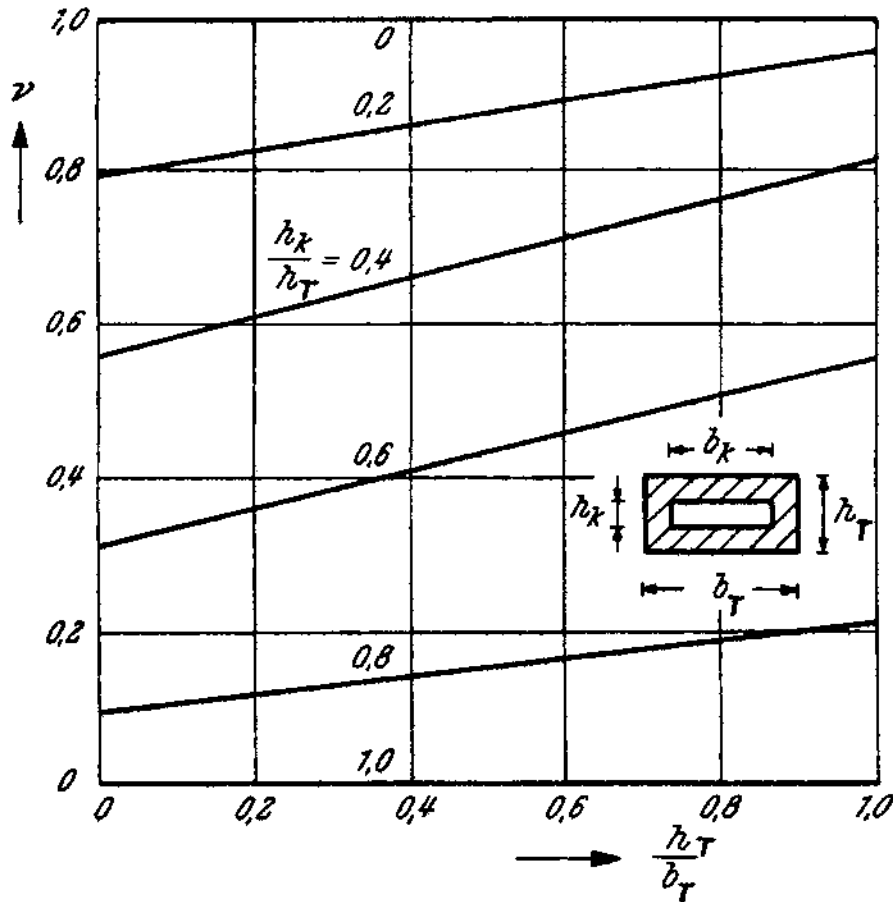


- Instead of parallel connection of two *Roebel* bars per slot a complete twisting of four rows of strands (rows I, II, III, IV) is possible, but very expensive, so rarely done !



# 3. Eddy current losses in winding systems

## Eddy current losses in hollow conductors (1)



- In *Roebel* bars with included hollow conductors for direct liquid cooling the conductor height  $h_T$  is increased due to the duct height  $h_k$  !
- But the copper cross section is the same as in massive strands without duct !
- For slot stray flux caused 2<sup>nd</sup> order eddy currents FIELD's formulas can be used, if the **hollow conductor coefficient**  $\nu$  is used :

$$k_m = 1 + \frac{m^2 - 0.2}{9} \xi^4 \nu$$

If all conductors are hollow !

$$\nu = \frac{12 \cdot J_T \cdot A_T}{b_T^2 h_T^4}$$

$$J_T = \frac{1}{12} \cdot (b_T h_T^3 - b_k h_k^3) \quad A_T = b_T h_T - b_k h_k$$

# 3. Eddy current losses in winding systems

## Eddy current losses in hollow conductors (2)

### Roebel bar with mixed massive and hollow conductors (strands):

#### Two layer winding:

$m_H/2$  : numbers of hollow strands in vertical direction per bar

$m_V/2$  : number of massive strands in vertical direction per bar

$m/2 = m_H/2 + m_V/2$  : total number of strands in vertical direction per bar

#### Simplified calculation:

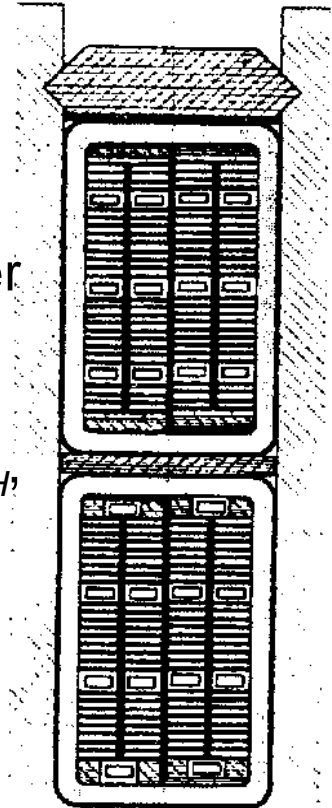
Current displacement of massive strands  $k_{mV}$  and hollow strands  $k_{mH}$  are regarded separately, as if all strands were hollow or massive:

$$k_{mV} = 1 + \frac{m^2 - 0.2}{9} \xi_V^4, \quad k_{mH} = 1 + \frac{m^2 - 0.2}{9} \xi_H^4$$

### Average increase of losses due to eddy currents in all strands:

$$k_{m,V+H} = k_{mV} \cdot \frac{m_V}{m} + k_{mH} \cdot \frac{m_H}{m}$$

$$m_H = 6, m_V = 24 \\ m = 30$$



# 3. Eddy current losses in winding systems

## 3.7 Three phase winding for super-conducting generators

- Wires with super-conductors consist of thin twisted super-conducting filaments, embedded in **copper or silver “matrix” conductor** ! In case of quenching (= super-conductor gets normal conducting due to overload) the resistance of the super-conductor material is so big, that the “matrix” must take over the current !
- **Rotor:** DC excitation: Super-conducting winding = no Ohmic losses !
- **Stator:** AC winding: Eddy current losses in “matrix” of the winding would yield too high losses, causing high amount of cooling power (e.g. 30 K above absolute zero !)

**So conventional copper is used for AC three-phase stator winding !**

- Big rotor excitation allows big air gap field of 1.5 ... 1.8 T, so stator teeth would be strongly saturated. Therefore teeth are omitted, an additional conductors are placed there, fixed by special non-conducting cylinder of glass fibre (“**air gap winding**”)!

Hence current loading  $A$  is increased by factor 2.

- **Increase of utilization:**  $C_{sc} / C_{norm} = (A_{sc} / A_{norm}) \cdot (B_{sc} / B_{norm}) = 2 \cdot 1.5 = 3$

**Super-conducting synchronous machine is only 1/3 in size at lower total losses !**

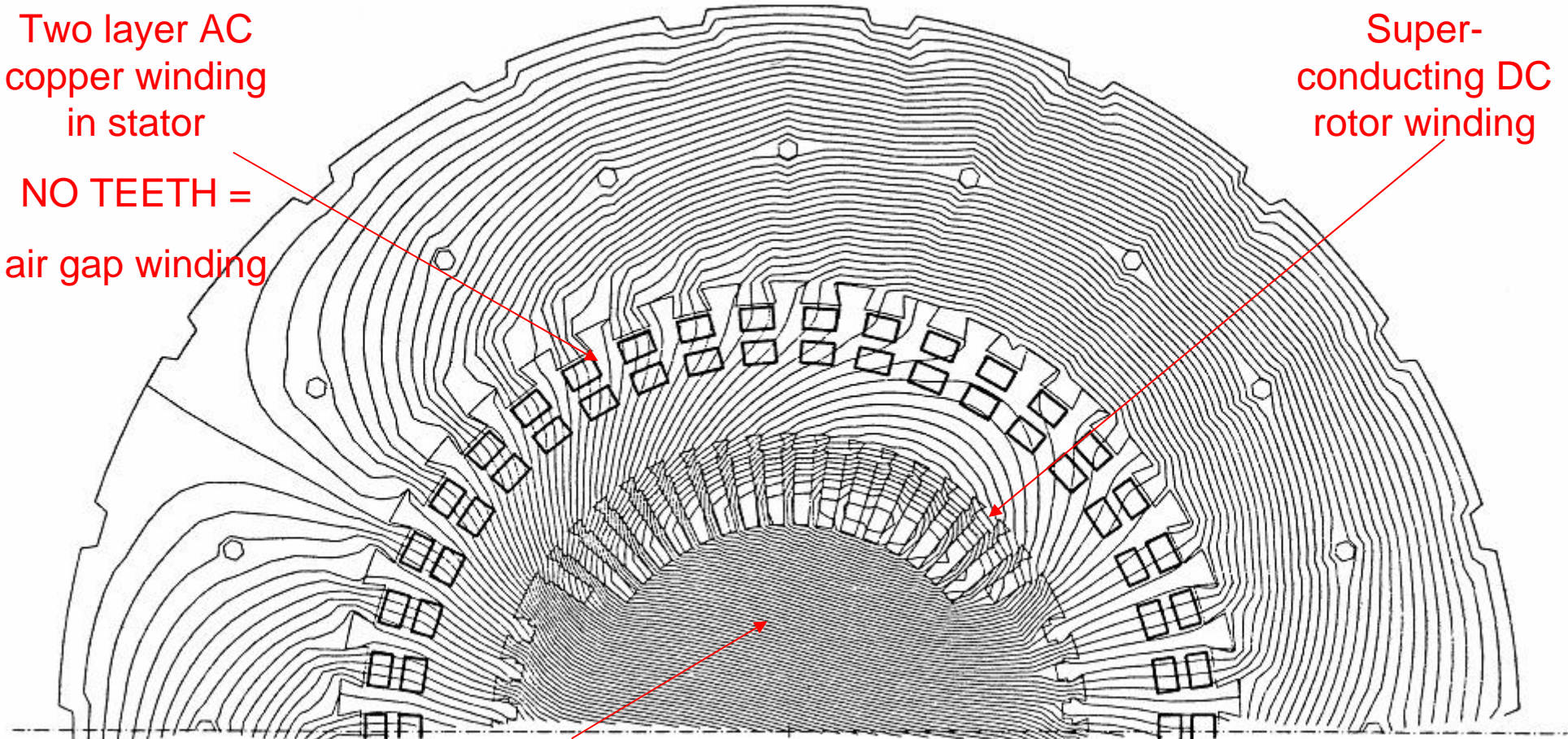
# 3. Eddy current losses in winding systems

## Three phase air gap winding for super-conducting generator

Two layer AC copper winding in stator

NO TEETH = air gap winding

Super-conducting DC rotor winding

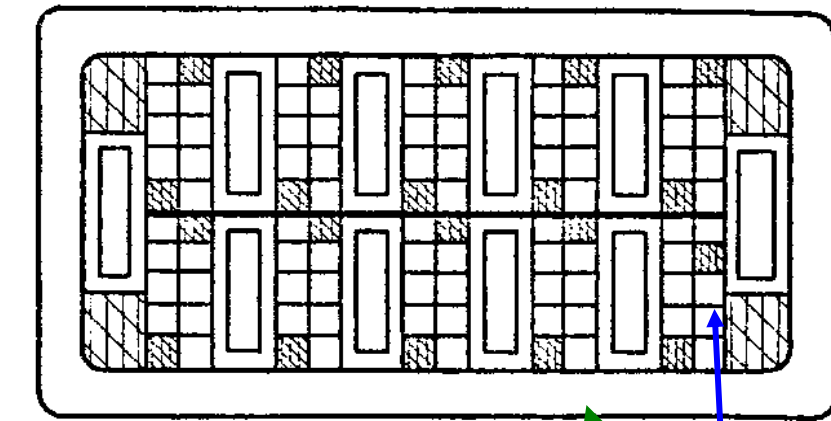


Usually also the rotor iron is made of non-magnetic (stainless) steel due to the high flux density !

# 3. Eddy current losses in winding systems

## Three phase winding for super-conducting generators

- *Roebel* bars of stator air gap windings are exposed to radial and transversal flux density  $B_r$  and  $B_q$  due to lack of flux guiding teeth !



One bar per layer

One small *Roebel* bar with transposed strands

$B_r$

$B_q$

- Therefore one bar per layer consist of e.g. 10 twisted and parallel connected small *Roebel* bars, which in their turn consist of e.g. 8 parallel and twisted strands !

- The small *Roebel* bars eliminate mainly the transversal flux linkage, whereas the bar itself eliminated the radial flux linkage !

