Price Updating with Production Networks*

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Version: March, 2019

Abstract

We provide a general non-parametric framework of variable markups in the presence of production networks. The framework does not impose structural assumptions on market environment, demand or production, and delivers elasticities that can be directly taken to the data. Our approach allows to estimate pass-through and firms' responses to environment's prices without the need to estimate marginal costs, but relies on observed firm-level price changes and input shares. Combined with the observation that firms are connected through buyer-supplier relationships, incomplete pass-through and price competition have profound implications for propagation and aggregation properties in terms of the exchange rate disconnect puzzle, monetary policy and productivity shocks.

Keywords: Pricing, production networks, pass-through, variable markups.
JEL codes: D21, L14, L16.

*We would like to thank Meredith Crowley, Emmanuel Dhyne, Catherine Fuss, Jozef Konings and participants at the NBB 2018 Colloquium for various comments and discussions. This research has been initiated in the context of the 2018 Colloquium of the National Bank of Belgium on "Understanding inflation dynamics: the role of costs, mark-ups and expectations".

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1 Introduction

Under constant markups, firms completely pass on variable cost shocks to their output prices. This complete pass-through is a feature of many canonical models of imperfect competition in both micro and macro. At the same time however, there is repeated empirical evidence that firms do not fully pass through these cost shocks, and that firms also strategically respond to other prices in their environment, allowing markups to vary.[1]

Under the observation that firms buy inputs from other firms and sell their output to other firms, the predicted accumulated effects of shocks are very different for complete or incomplete pass-through regimes. With complete pass-through, any shock to input prices or productivity is ultimately fully borne by the final consumer, appropriately weighted by the particularities of the production network. With incomplete pass-through however, cost shocks will decay as they pass through more firms.[2]

Consider the following simple thought experiment of an economy with n firms on a line, each only selling to its direct neighbor to the right. Suppose a shock affects the costs of the first firm. With complete pass-through, this shock is fully reflected in the output price of the last firm, even as n grows to infinity. With incomplete pass-through however, this shock decays at a rate of β^n. Even with relatively high pass-through rates (e.g. $\beta = 0.6$) and few firms ($n = 4$), the change in output price of the last firm would only be 0.13, or almost an order of magnitude smaller than the initial shock.

In this paper, we argue that this simple observation has important implications for propagation and aggregation. First, this mechanism might provide an alternative explanation for the well-known exchange rate disconnect puzzle. The puzzle states that the large observed dispersion in import price changes cannot be matched with a lower observed dispersion in consumer prices. Most existing solutions suggest nominal rigidities to explain this discrepancy. However, this mapping from import prices to consumer prices might also be explained by propagation of incomplete pass-through under flexible prices. Second, a standard exercise in the network propagation literature considers the effect of a productivity shock on aggregate outcomes. These shocks are then weighted by the importance of the firm in the economy to obtain their aggregate impact. However, under incomplete pass-through, the aggregation properties are different.

We first propose a general, non-parametric framework of variable markups with buyer-supplier relationships. Firms produce output according to a non-parametric cost function, combining inelastic factors with inputs from other firms. The framework allows for variable markups at the firm level with minimal restrictions on costs, possible price setting mechanisms, production technologies and product-market competition. In particular, firms are static cost minimizers and the cost function exhibits constant returns to scale with respect to variable inputs, while non-constant returns to scale at the firm-level are allowed through fixed costs of production. Under these conditions, the resulting pricing equation is consistent with a large class of mechanisms, including but not limited to profit maximization, cost-plus pricing and revenue maximization. Moreover, markups charged by the firm are not necessarily an equilibrium outcome such as a strategic best response function across oligopolistic competitors, but are also consistent with models in which the firm is a price follower in its sector or in the aggregate.

The framework delivers elasticities on how costs and markups respond to shocks in input prices, productivity and environment's prices, consistent with a broad class of price setting models and functional forms. We estimate these elasticities exploiting rich data on firm-to-firm linkages and domestic producer prices for Belgium. Importantly, we do

[1] Typical settings include exchange rate pass-through in international macro (e.g. Atkeson & Burstein (2008); Burstein & Gopinath (2014) and more generally Amiti et al. (2016)), selected industries in industrial organization (e.g. Goldberg & Verboven (2001); Nakamura & Zerom (2010); Fabra & Reguant (2014)), and tax incidence (e.g. Weyl & Fabinger (2013)).

[2] See e.g. Domar weights in Hulten (1978); Gabaix (2011), the influence vector of Acemoglu et al. (2012), or more intricate centralities as in Baqaee (2018).

not need to estimate marginal costs or markups. Changes in marginal costs are additively separable in (i) shocks to the input price index and (ii) productivity shocks. We obtain (i) directly as a weighted average from observables on input shares and input price shocks, and (ii) as changes in estimated quantity-based productivity, $TFPq$, using a production function approach that also exploits this heterogeneity in firm-level input price indices.[3]

Estimating the pricing equation however, implies dealing with various endogeneity issues. First, both input price changes and other firms' price changes can be simultaneously determined with changes in output prices. Second, there is possible measurement error in the regressors, as prices are obtained from unit values. Third, selection bias might arise if selection occurs on unobservables. We first exploit the directed structure of the production network to attenuate the simultaneity problem, and obtain a directed acyclical graph, much in the spirit of Pearl (2000). We also provide empirical evidence that firms do not systematically add or drop suppliers in response to price shocks, and that there is no correlation between the magnitude of the input shares and changes in input price shocks. For the remaining endogeneity issues, we employ an instrumental variable approach, and use average prices and productivity shocks to construct five instruments from the different micro datasets. Identification rests on heterogeneity in exogenous input shares, non-transitivity and the large sparse matrix characteristics of the directed production network.[4]

The price updating results are telling: on average across all firms, the cost pass-through of input price shocks to output prices is incomplete and well below one, with an estimated coefficient of 0.52: a 1% increase in input prices leads to a 0.52% increase in output prices. The environment's price elasticity is also highly significant and positive at 0.38. This confirms that empirically, models of price setting behavior with constant markups, such as perfect competition or monopolistic competition with CES preferences are refuted, at least at the average. These results are not particular to import price shocks or small open economies, but show that variable markups are a fundamental feature of firms' pricing decisions. Importantly, these variable markups have consequences for propagation and aggregation, even in closed economies.

Next, we show that our estimated elasticities of pass-through and firms' responses to their environment have profound implications for propagation and aggregation. The total effect of an initial shock on the distribution of firm-level prices and the producer price index ultimately depends on (i) the amount of pass-through and firms' best responses, (ii) the network structure of production and the weights given to other prices in the firm's environment, and (iii) the nature of the shock.

Finally, in order to combine both international trade data and domestic production data, we have developed a detailed concordance procedure that takes into account changes in the statistical classifications of the Combined Nomenclature (CN) and Prodcom (PC). These classifications tend to change regularly over time, and for various reasons, which makes tracing products over time non-trivial. Our procedure takes into account these changes, including not only one-to-one correspondences, but also non-singular correspondences (i.e. many-to-one, one-to-many and many-to-many) from one year to another. Our procedure differs from other concordance methods, most importantly in the sense that we do not need to aggregate products into "family trees" (Pierce & Schott (2012)) over time. Instead, we identify price changes for products between any two years, and do not impose a panel structure over time. This procedure is applied to both the CN and PC datasets over time, as well as a contemporaneous correspondence between CN and PC.

This paper connects to different strands of literature. First, the paper is related to the theoretical literature on incomplete pass-through and variable markups (e.g. Atkeson & Burstein (2008); Melitz & Ottaviano (2008); Weyl &

[3] Standard estimates of TFP rely on revenue-based or value-added production function estimations, generating simultaneity in prices in our main regression. Therefore, we estimate a quantity-based measure of productivity (*TFPq*), which purges the TFP measure from prices. We do not rely on sector-level price deflators for materials, but construct the change in firm-level input price indices from the micro data.

[4] This is a reformulation to a directed and weighted graph of the arguments in Bramoulle et al. (2009); De Giorgi et al. (2010), who consider the identification problem in the linear-in-means peer effects model of Manski (1993).

Fabinger (2013); Atkin & Donaldson (2015); Edmond et al. (2015); Amiti et al. (2016); Parenti et al. (2017); Arkolakis & Morlacco (2018)). We provide a very general and non-parametric framework of price setting that is consistent with a broad class of models of competition and demand. Importantly, variable markups in our set-up can arise from strategic interaction between firms, but also from non-strategic responses of firms to their environments, such as aggregate price indices or even simply following the market price evolution.

Second, this paper relates to the empirical literature on variable markups and the estimation of incomplete pass-through. For instance, many contributions from the international macro literature have focused on how international shocks such as exchange rate movements, affect domestic prices through importers (for an overview, see Burstein & Gopinath (2014)). Conversely, many papers have estimated incomplete pass-through and variable markups in specific industries, such as coffee (Nakamura & Zerom (2010)), beer (Goldberg & Hellerstein (2013)), cars (Goldberg & Verboven (2001)), or electricity (Fabra & Reguant (2014)). Exploiting the rare features of the various datasets, we estimate pass-through rates and adjustment of firms' prices to their environment's price index across all manufacturing sectors, including non-importing firms which play a crucial role in our setup.

Third, this paper speaks to a growing literature on production networks, pricing and propagation (e.g. Magerman et al. (2016); Baqaee (2018); Baqaee & Farhi (2018)). However, most models impose perfect pass-through, even under double marginalization, and any shock ultimately ends up unattenuated at the final consumer. This has important aggregation implications, as changes in both producer and consumer surplus are accrued over the whole network of production. Some notable exceptions with incomplete pass-through include Kikkawa et al. (2018); Grassi (2018); Heise (2018).

The remainder of this paper is organized as follows. Section 2 describes the non-parametric model of pricing. Section 3 discusses identification and estimation and Section 4 presents the pass-through results. Section 5 performs the counterfactual analysis, while Section 6 concludes.

2 A general framework of price updating

We present a general framework of how producers change output prices in the presence of production networks. The framework accommodates variable markups at the firm level with minimal restrictions on price setting mechanisms, product-market competition and production technology. The framework also allows to recover elasticities on costs, prices and markups that are consistent with a broad class of widely used functional forms.

2.1 The network structure of production

There are N producers indexed by j, who produce output combining fixed inputs with variable inputs from other producers, and sell their output to other producers and final demand. Producer j produces at time t, according to the following cost function

$$y_{jt}c_{jt}\left(\left(1+\tau_{ij}\right)p_{1t},...,\left(1+\tau_{ij}\right)p_{nt},z_{jt}\right)+F_{jt}$$

where y_{jt} is the output level, τ_{ij} is a pair-specific friction on input i used by j, p_{it} is the price of input i, z_{jt} is producer j's efficiency in transforming inputs into outputs and F_{jt} is a fixed cost to be paid in every period t.

Some notes. First, c_{jt} is a non-parametric function accommodating any production function as long as production exhibits constant returns to scale with respect to variable inputs. Second, this specification embeds the network structure of production: i's output is used as an input by producer j, whose output can be used again as an input by other producers. Third, bilateral frictions can drive a wedge between the output price of supplier i and the input price faced by buyer j. These ad valorem frictions are very general conditional on being fixed at the pair level, and can include value-added taxes, transport costs, tariffs etc. Fourth, this specification does not impose a particular form of technological change z_{jt}, and is consistent with for instance Hicks- and Harrod-neutral productivity, but also allows for non-neutral technological change.[5] Finally, in the main text we consider single-product firms for ease of exposition, and Appendix C presents a multi-product version of the model with additional identification assumptions.

2.2 Pricing and markups

Next, consider the following pricing equation of producers j at time t, which holds under static cost minimization:

$$\ln p_{jt} = \ln c_{jt}\left(\left(1+\tau_{ij}\right)p_{1t},...,\left(1+\tau_{ij}\right)p_{nt},z_{jt}\right) + \ln \mu_{jt}\left(p_{jt},\mathscr{P}_{-jt};\xi_{jt}\right) \tag{1}$$

where p_{jt} is the output price of producer j, and μ_{jt} represents the markup of j, which is a function of j's own price p_{jt}, an index of prices in j's environment $\mathscr{P}_{-jt}$, and exogenous shifters $\xi_{jt} = (\xi_{1t},...,\xi_{lt})$.[6] These perceived demand shifters capture any source of output price variation that is not systematically related to marginal costs or to the environment's price index.

This pricing equation is consistent with a large class of price setting mechanisms. First, eq(1) does not impose profit maximization, but is consistent with other pricing schemes such as cost-plus pricing or revenue maximization.[7] Second, the pricing equation does not impose any particular market structure or demand system, and allows for either no markups, constant markups, or variable markups.[8] Third, the index of prices in j's environment $\mathscr{P}_{-jt} = \left\{(p_{1t},...,p_{lt}),(\lambda_{1jt},...,\lambda_{ljt}) : \sum_{l=1}^{L_j}\lambda_{ljt} = 1\right\}$ with weights λ_{ljt} is very general, and the exact construction of the index depends on the underlying model of price setting behavior. The relevant set of firms could include the price of direct competitors, either under simultaneous move games or sequential price setting, but also geographically close firms, sector-level or aggregate price indices etc., or any combination of these.[9] Moreover, $\ln \mu_{jt}\left(p_{jt},\mathscr{P}_{-jt};\xi_{jt}\right)$ does not have to be an equilibrium outcome such as a strategic best response scheme across oligopolistic competitors, but is also consistent with off-equilibrium or non-strategic price setting behavior (e.g. producers that update prices in response to aggregate prices only). Hence, eq(1) can also be seen as a reduced form pricing equation, in which the nature of price setting is left unspecified. Finally, the exogenous shifters ξ_{jt} are unconstrained in our setting. If we would further restrict the model by assuming profit maximization and pinning down market structure and demand, ξ_{jt} captures quantity effects such as the price elasticity of demand.

This general setting implies we can obtain estimates for these elasticities from the data without *ex ante* enforcing

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[5] Consider for example a non-neutral technology such that a shock would affect only the productivity of one particular input but not that of others. At the cost minimizing point, the marginal cost of that input equals the marginal revenue. Then, the gains from increased productivity for this particular input is just given by its relevant share in production.

[6] $j \in L_j = \{1,...,l\}$ denotes the set of relevant firms in j's environment that might influence j's markups. The set of relevant firms in j's environment excluding j is denoted as $-j$.

[7] Under cost-plus pricing, producers set prices as pre-determined markups over average variable costs or marginal costs, without taking into account competition or demand for that good. Hence, prices are not profit maximizing while the firm might still be cost minimizing.

[8] For example, the environment price index accommodates models of oligopolistic competition (either Cournot or Bertrand) with nested CES demand (e.g. Atkeson & Burstein (2008)). The framework also encompasses models of monopolistic competition with non-CES preferences, such as linear demand systems (e.g. Melitz & Ottaviano (2008)), Kimball (1995) preferences, or when firms adjust their markup in response to sector-level or aggregate prices (e.g. Parenti et al. (2017)). Alternatively, variable elasticity of demand systems (VES) can generate variable markups under different market structures (e.g. Dornbush (1987), Arkolakis & Morlacco (2018)). Yet another setup allows for CES demand with additive distribution costs as in Corsetti & Dedola (2005). Under monopolistic competition with CES preferences, the workhorse model in international economics (e.g. Melitz (2003)), markups are constant. Trivially, under perfect competition markups are zero. For a more formal discussion, see e.g. Burstein & Gopinath (2014) or Mrázová & Neary (2017).

[9] For example, for a broad class of models, a sufficient statistic for λ_{ljt} is the market share of competitor l in j's product market. Then $\mathscr{P}_{-jt} = \sum_{l=1}^{L_j}\lambda_{ljt}p_{lt}$ is a weighted average of prices of competitors. More generally, $\mathscr{P}_{-jt}$ can even be non-linear in prices.

particular models of competition, demand or production technologies.

2.3 Price updating in production networks

Next, totally differentiating eq(1) leads to the following decomposition of the pricing equation:

$$d\ln p_{jt} = \underbrace{\sum_{i\in S_{jt}} \frac{\partial \ln c_{jt}}{\partial \ln p_{it}} d\ln p_{it}}_{\text{total input price shock}} + \underbrace{\frac{\partial \ln c_{jt}}{\partial \ln z_{jt}} d\ln z_{jt}}_{\text{productivity shock}} + \underbrace{\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}} d\ln p_{jt}}_{\text{own price markup effect}} + \underbrace{\frac{\partial \ln \mu_{jt}}{\partial \ln \mathscr{P}_{-jt}} d\ln \mathscr{P}_{-jt}}_{\text{environment price index effect}} + \underbrace{\frac{\partial \ln \mu_{jt}}{\partial \xi_{jt}} d\xi_{jt}}_{\text{demand shifters}} \tag{2}$$

where S_{jt} is the set of suppliers to j. A change in j's output price is thus a combination of (i) a change in input prices p_{it}, (ii) a productivity shock to j's technology z_{jt}, and (iii) a change in markups $\mu_{jt}(p_{jt}, \mathscr{P}_{-jt}; \xi_{jt})$.

Eq(2) merits some explanation. First, the total input price shock evaluates how changes in input prices affect the cost function. It is a linear combination of shocks to all input prices p_{it} to j, and reflects how j's cost function responds to all these input price shocks combined. This implies that shocks to all input prices can be linearly aggregated, independent of the exact functional form of the cost function. Moreover, the cost response to an input price shock can be written as

$$\frac{\partial \ln c_{jt}}{\partial \ln p_{it}} = \frac{p_{ijt}x_{ijt}}{\sum_{i\in S_{jt}} p_{ijt}x_{ijt}} = \omega_{ijt} \tag{3}$$

where $\sum_{i\in S_{jt}} p_{ijt}x_{ijt}$ is j's total variable cost, and ω_{ijt} is the elasticity of the marginal cost with respect to a change in one input price p_{it}. The second equality uses Shephard's lemma to equate the input elasticity to the share of expenditures on input i. Next, it is straightforward to aggregate individual input price shocks to a change in the input price index for producer j:

$$d\ln P_{jt} \equiv \sum_{i\in S_{jt}} \omega_{ijt} d\ln p_{it} \tag{4}$$

The total change in j's input price index, $d\ln P_{jt}$, is a weighted average of price shocks to inputs i bought by j, weighted by their share in total variable costs ω_{ijt}. It is important to stress that eq(4) is not restricted to a Cobb-Douglas input price index, but is consistent with several functional forms, since individual price shocks can be aggregated linearly to a total input price shock, independent of the underlying functional form of the cost function or the input price index.

Second, eq(2) separates the impact of input price shocks from that of productivity shocks within the cost function. Again from an envelope theorem argument, at the cost minimizing input tuple $(x_{1jt}, ..., x_{njt})$, any potential input reallocation effects through $d\ln x_{ijt}$ have no effect on marginal costs, and therefore also not on output prices. Hence, the total impact of a productivity shock on marginal cost is given by $\frac{\partial \ln c_{jt}}{\partial \ln z_{jt}} d\ln z_{jt} = \frac{\partial \ln y_{jt}(x_{1jt},...,x_{njt},z_{jt})}{\partial \ln z_{jt}} d\ln z_{jt}$.

Third, the markup adjustment is a combination of change in j's own price p_{jt}, that of j's environment price index $\mathscr{P}_{-jt}$ and exogenous shifters ξ_{jt}. The first markup effect isolates the own price effect on j's markup. This elasticity directly relates to the amount of cost pass-through, as will be shown below. The environment's price index effect evaluates how j adjusts its markup in response to its output environment, while the last term captures the markup response to all price variation not due to marginal costs or the environment's price index.

Finally, rearranging eq(2) and using eq(3) generates an estimation equation that can be taken to the data:

$$d\ln p_{jt} = \beta_{jt} \sum_{i\in S_{jt}} \omega_{ijt} d\ln p_{it} + \gamma_{jt} d\ln z_{jt} + \delta_{jt} d\ln \mathscr{P}_{-jt} + \eta_{jt} d\xi_{jt} \tag{5}$$

where the coefficients have a clear structural interpretation:

$$\begin{cases} \beta_{jt} \equiv \frac{1}{1-\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \\ \gamma_{jt} \equiv \frac{1}{1-\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \frac{\partial \ln y_{jt}}{\partial \ln z_{jt}} \\ \delta_{jt} \equiv \frac{1}{1-\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \frac{\partial \ln \mu_{jt}}{\partial \ln \mathscr{P}_{-jt}} \\ \eta_{jt} \equiv \frac{1}{1-\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}}} \frac{\partial \ln \mu_{jt}}{\partial \xi_{jt}} \end{cases} \tag{6}$$

First, β_{jt} can be interpreted as a cost pass-through parameter under very general conditions and is consistent with various models of firm behavior. It captures how much a change in input prices p_{it} relates to a change in output price p_{jt}, and nests several pricing mechanisms. Under either no or constant markups, $\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}} = 0$ and $\beta_{jt} = 1$, and so cost pass-through is complete. Under variable markup regimes however, $\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}} \neq 0$, and thus $\beta_{jt} \neq 1$. Whether $\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}} \lesseqgtr 0$ depends on the specification of $\mu_{jt}(p_{jt}, \mathscr{P}_{-jt}; \xi_{jt})$, but most price setting models with variable markups will generate $\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}} < 0$, such that $\beta_{jt} < 1$ and incomplete pass-through occurs.[10] Second, as is clear from eq(6), this pass-through parameter also governs the impact of all other shocks that influence changes in output prices, by rescaling the relevant elasticities by a factor β_{jt}. Third, given consistent estimates on all parameters, we can back out the different underlying elasticities.

3 Identification and estimation

We exploit the rich structure of the different datasets described in Appendix A to obtain consistent estimates for the structural parameters $\theta = (\beta, \gamma, \delta)'$ in eq(5).

3.1 Variables

Estimating eq(5) requires information on changes in output prices $d \ln p_{jt}$ and input prices $d \ln p_{it}$, input shares ω_{ijt}, efficiency shocks $d \ln z_{jt}$, changes in environment's prices $d \ln \mathscr{P}_{-jt}$ and perceived demand shocks $d\xi_{jt}$. We discuss each of these in more detail.

First, changes in output prices $d \ln p_{jt}$ are obtained as log differences in unit values from year $t-1$ to t for all producers j in the Prodcom dataset. The model in Section 2 presents a general framework on how single-product producers update prices in the presence of production networks. When presenting the empirical results below, we report regression results for the main PC8 product of producer j, trivially including single-product firms.[11] In Appendix C, two extensions of the pricing equation for multi-product firms are derived, which induce some additional identification assumptions. Under these additional assumptions, the parameter estimates for $\theta = (\beta, \gamma, \delta)'$ are very similar to the baseline estimates.

Second, input prices $d \ln p_{it}$ are constructed as log differences in unit values for domestic suppliers and imports. For imports, $d \ln p_{it}$ is simply the change in the import price faced by j (by country-CN8). For domestic suppliers in the Prodcom data, the output price change of supplier i is used as the input price paid by j. For this subset of suppliers, it

[10] Under particular conditions, pass-through rates larger than one can be obtained. E.g. with consumer heterogeneity, higher prices can force consumers with high demand elasticity to leave the market, causing the average demand elasticity to fall and markups to rise (Burstein & Gopinath (2014)).

[11] Across all multi-product firms in the sample, the main product accounts for 66% of total revenues on average.

is necessary to assume that producer j buys the full bundle of products of i if i is a multi-product firm. We then use the change in the firm-level output price index (a weighted average of price changes across all PC8 products of producer i) to construct $d\ln p_{it}$. For domestic suppliers not in Prodcom (which mostly contain services inputs) we observe the main NACE activity of the firm, and we use the sector-level output deflator of firm i to proxy for $d\ln p_{it}$. Finally, in very few cases, the output deflator of the supplier is also not observed. Then, the sector-level input deflator of j is used instead. It is important to underline that in most standard firm-level datasets and applications (e.g. production function estimation), only this latter sector-level input deflator is observed and used as a proxy for changes in input prices. We use firm-level output prices of suppliers in the majority of the cases, and in all cases, changes in prices are correctly weighted by the firm-pair level input shares before being aggregated to an input price index, generating firm-level variation in input prices based on the particular set of suppliers to j and their respective input shares.

Third, input shares ω_{ijt} for all suppliers i to j are directly observed in the data. Input shares are constructed as $\omega_{ijt} = \frac{m_{ijt}}{\sum_i m_{ijt}}$, where m_{ijt} is the value of expenditures of producer j on inputs sourced from i. For domestic suppliers, i refers to a firm, while for imports, it refers to a product-country observation. Even when supplier prices are missing, input expenditures are always observed at the pair level. This allows us to construct the true input shares for every supplier to j, which is necessary for Shephard's lemma to hold. We use lagged input shares, ω_{ijt-1}, when estimating the pricing equation, to ensure weak exogeneity and avoid measurement issues with weights being influenced by contemporaneous changes in prices.[12] In the baseline estimations, capital inputs and labor are considered to be part of fixed costs, and do not enter the input shares.[13] Robustness tests with labor being part of the marginal cost bundle produce very similar results (not reported).

Next, $\ln z_{jt}$ is estimated as the residual of a production function using the GMM setup as in Wooldridge (2009). Our procedure is similar to the productivity literature (e.g. De Loecker & Warzynski (2012), Ackerberg et al. (2015)), but with some marked differences exploiting the rich structure of the data. The micro datasets allow us to overcome several typical measurement problems, which in turn improve identification of the resulting TFP estimates. First, exploiting the Prodcom data, the production function is estimated in quantities for all producers j, generating a measure of $TFPq$, rather than the more commonly estimated revenue- or value added-based measures. This is crucial, as (i) the TFP measure is purged from prices, avoiding potential simultaneity issues when estimating eq(5), and (ii) output quantities are not derived from a combination of firm revenues and sector-level output price deflators.[14] Second, TFP estimation does not rely on input price deflators as is standard in this literature. Instead, $d\ln P_{jt}$ is constructed directly from information on supplier prices and their input shares as described above. This implies that producers face firm-specific input prices for their input bundles, which take into account heterogeneity in sourcing patterns and prices paid for those bundles, rather than sector-level prices resulting from deflators. Note that, while the pricing schemes in our framework are very general, a few more assumptions are needed on the underlying production function to estimate productivity using the existing machinery of the productivity literature. These include some restrictions on the admissible functional forms and timing assumptions on how firms choose variable and fixed inputs. We want

[12] While using lagged input shares implies only price changes for continuing products from $t-1$ to t are potentially identified, this is a mild constraint in the data: across all producers, the average input expenditure share of continuing products is over 90% (see Figure 3). Taking into account changes on the extensive margin of the input product mix would imply estimating shadow prices for unobserved products, forcing us to take a stance on price setting behavior and functional forms.

[13] Particularly in Belgium, hiring and firing is not flexible, and many wages are subject to indexation schemes that are linked to inflation. Therefore, as a baseline, we consider labor to be part of fixed costs, to be paid by j at the start of every period t. Note that hired labor through temporary employment agencies (NACE 7820) is recorded as intermediary inputs in our data, and is part of variable costs. Hence labor can be split up into a fixed and variable part, where the hiring of temporary workers scales more easily with the level of production in the short term.

[14] This is effectively borne out in the pass-through estimations. *TFPq* remains very stable across specifications, and is not systematically correlated with any of the other variables. Moreover, the sign of the coefficient is negative, as to be expected: an increase in productivity is related to a decrease in output prices, all else constant. Alternatively estimating the pass-through regressions with revenue-based TFP, which is not purged from price effects, generates a positive estimated coefficient and induces simultaneity.

to stress that these assumptions are only imposed to estimate $\ln z_{jt}$, and do not affect the generality of the pricing equation.

Fifth, for exposition we follow Amiti et al. (2016), and operationalize $d\ln \mathscr{P}_{-jt}$ as the market share weighted average of price changes of other producers in the same product category. In particular,

$$d\ln \mathscr{P}_{-jt} = \sum_{l\neq j\in PC_{jt}} \lambda_{l\,jt-1} d\ln p_{lt} \tag{7}$$

where PC_{jt} is the set of firms l in the same PC 4-digit product category as producer j at time t, and $\lambda_{l\,jt-1}$ is the lagged market share of competitor l in that sector.[15] Note that this is just an empirical choice. Generally, $d\ln \mathscr{P}_{-jt}$ depends on the specific assumption made on market structure and demand, which we have not pinned down in the pricing framework. Alternatively, one can construct an environment's price index based on assumptions of the underlying model of competition, including responses of j with respect to aggregate price indices, geography, or any definition of relevant markets in general.

Finally, we use fixed effects to capture the unobserved demand shifters $d\xi_{jt}$. We operationalize $d\xi_{jt}$ as a year fixed effect $d\xi_t$, a sector fixed effect $d\xi_s$, and a sector-time fixed effect $d\xi_{st}$.

3.2 Identification issues

The structural estimation equation is given by

$$d\ln p_{jt} = \beta \sum_{i\in S_{jt}} \omega_{ijt-1} d\ln p_{it} + \gamma d\ln z_{jt} + \delta \sum_{l\neq j\in PC_{jt}} \lambda_{l\,jt-1} d\ln p_{lt} + \eta d\xi_{st} + \varepsilon_{jt} \tag{8}$$

Estimating the structural parameters $\theta = (\beta,\gamma,\delta)'$ in eq(8) by means of OLS will lead to biased and inconsistent estimates in the presence of endogeneity, in which case estimation of the causal effects of the regressors on changes in output prices and subsequent relevant policy predictions are elusive.[16]

There are several potential sources of endogeneity in eq(8). First, simultaneity arises if changes in output prices $d\ln p_{jt}$, changes in input prices $d\ln p_{it}$ and/or changes in competitors' prices $d\ln p_{lt}$ are jointly determined. This can be due to (i) cyclicality of the production network (when j also supplies its supplier i), and (ii) co-movement of prices across firms (e.g. through a best response function in an oligopolistic setting). While we cannot directly deal with (ii) in the data, and we resort to an instrumental variables strategy to that end, we exploit the directed structure of the production network to attenuate (i) by dropping cyclical input relations and (ii) suppliers i to j that are also competitors to j (i.e. in the same PC4 product category).[17]

Second, while we exploit the rare features of the datasets on micro prices, input shares and firm-product sales when constructing $d\ln P_{jt}$ and $d\ln \mathscr{P}_{-jt}$, there remains potential measurement error in the regressors. In particular, changes in prices are obtained from unit values, which are potentially a noisy measure of true but unobserved prices.

[15] In particular, $\lambda_{l\,jt} = \frac{\text{sales value}_{lkt}}{\sum_{l\in PC_{jt}} \text{sales value}_{lkt}}$ where k is the same 4-digit PC code to which j belongs, and such that $\sum_l \lambda_{l\,jt} = 1$. This implies that we allow for multi-product firms to be defined as a competitor only for this particular PC4 product, such that we do not erroneously allocate firm-level sales values (i.e. other products) to these market shares. For competitors l that are not in Prodcom, we use other firms in the same NACE 4-digit category as j.

[16] For instance, does the covariance between $d\ln p_{jt}$ and $d\ln p_{it}$ reflect true spillovers, or simply heterogeneity in $d\xi_{st}$ and ε_{jt} (i.e. homogeneous or heterogeneous responses to shocks)? If spillovers exist, this has implications for policy analysis, as (i) the identity of suppliers i matters, and (ii) the realized network structure of production affects policy choices and outcomes (see Bramoulle et al. (2009); De Giorgi et al. (2010) for a further discussion).

[17] As prices tend to be positively correlated, OLS estimates of β are expected to be biased downwards due to simultaneity. Comparing OLS estimates with and without this cleaning in Appendix D indeed confirms the direction of the bias.

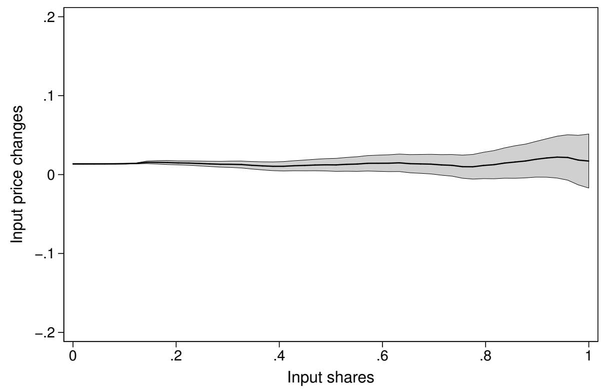


Figure 1: Covariance ω_{ijt-1} and $d\ln p_{it}$.

Additionally, we resort to the use of deflators for some observations. Since it is not obvious that this is classical measurement error, the direction of the bias of OLS estimates is unclear.

Third, sample selection induces bias if selection occurs on unobservables. A key assumption we make is that relationships are formed before the realization of shocks, and do not change in response to these shocks. As noted above, over 90% of the value of inputs is continuing between any two years, indicating stable relationships between suppliers and buyers. Moreover, on the intensive margin, there is no systematic correlation between the size of the input share and input price shock, as shown in the local polynomial plot in Figure 1. This pattern holds across suppliers within firms and also across firms within PC8 products. However, producers j might still add or drop suppliers in response to (un)favorable input price shocks of potential suppliers i. For selection on the extensive margin to generate bias in estimation, it must be that the distribution of $d\ln p_{it}$ is different for continuing suppliers versus added or dropped suppliers. I.e. we expect larger increases in prices for dropped suppliers than for continuing suppliers, and that new suppliers to j have experienced price drops such that they become attractive input suppliers to j. Figure 2 shows the distribution of price changes for continuing, added and dropped suppliers between any two years. There is no systematic difference in price changes across these types of suppliers, suggesting that firms do not systematically add or drop suppliers in response to realized price shocks of suppliers. Finally, it's worth mentioning that firms can match with favorable suppliers in levels (e.g. very productive or having a low price) without biasing the estimated coefficients, as the estimation equation is in changes, not levels. It would have to be the case that firms match based on expectations of $d\ln p$ or $d\ln z$.

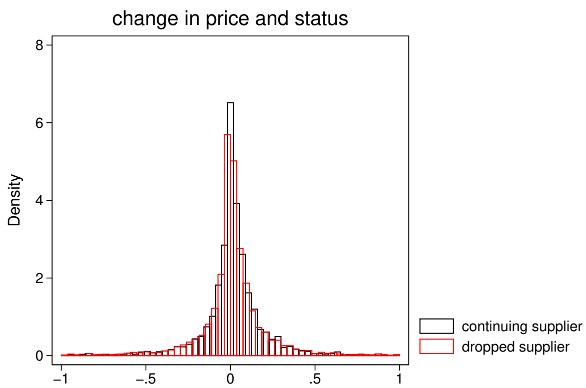
3.3 Instrumental variables

To account for these different endogeneity issues and to obtain consistent estimates for $\theta = (\beta, \gamma, \delta)'$, we implement an instrumental variable approach. We construct five instrumental variables for $d\ln p_{it}$ and $d\ln \mathscr{P}_{-jt}$.

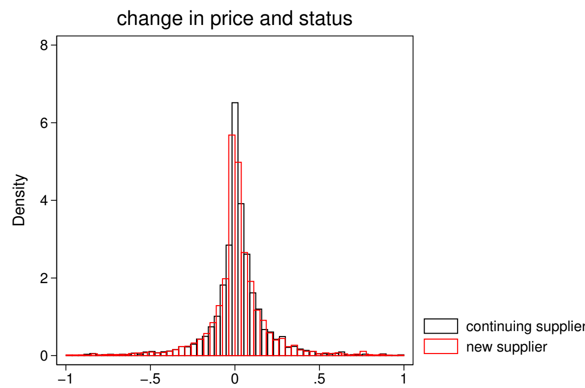
First, we use observed productivity shocks of suppliers i to instrument for $d\ln p_{it}$ of firms that are in the Prodcom dataset. In particular

$$d\ln P_{jt}^{IV1} \equiv \sum_{i\in S_{jt}} \omega_{ijt-1} d\ln z_{it}$$

where each supplier is weighted by its input share to j. The exclusion restriction implies that $\mathbb{E}(d\ln \mathbf{z}\varepsilon) = 0$. Intuitively, in the light of the directed acyclical nature of the production network, productivity shocks to supplier i only affect output prices of producer j through the input prices $d\ln p_{it}$. More formally, the reduced form of eq(8) (see Section 5)



(a) Continuing vs dropped.



(b) Continuing vs added.

Figure 2: Distribution of price changes across continuing, added and dropped suppliers.

shows that $d\ln z_{it}$ is a plausible candidate for an instrument to input prices.

Second, changes in input prices $d\ln p_{it}$ are also instrumented using a shift-share approach as in Freeman (1975, 1980); Bartik (1991).[18] We follow Goldsmith-Pinkham et al. (2018) and obtain identification under exogeneity of lagged input shares ω_{ijt-1}. In particular, the following instruments are derived from the next two accounting identities:

Definition 1. (i) The change in the input price index of j is a weighted sum of input shares and input price shocks, i.e. $d\ln P_{jt} = \sum_{i\in S_{jt}} \omega_{ijt-1} d\ln p_{it}$; (ii) input price changes can be decomposed as $d\ln p_{it} = d\ln \bar{p}_{it} + d\ln \tilde{p}_{it}$, where $d\ln \bar{p}_{it}$ is the average price change within a particular product category, and $d\ln \tilde{p}_{it}$ is the idiosyncratic variation around that average price.

The first part just follows from the definition of the change in the input price index, while the second part constructs the average (or aggregate) price change to be used in the instrument. We construct two instruments based on this definition: one instruments for imported inputs, grouped within a CN4 category, and a second one for domestic inputs, grouped within a PC4 category. We use a weighted average to construct $d\ln \bar{p}_{it}$, with weights given by sales shares of each observation, and we use a leave-one-out mean to avoid mechanical correlation between i and the instrument,[19] such that

$$d\ln \bar{p}^{CN}_{-it} = \sum_{k\neq i\in CN4} \lambda_{kit-1} d\ln p_{kt}$$

and similarly for the domestic inputs at $PC4$ level. Hence, the resulting instruments are

$$d\ln P^{IV2}_{jt} = \sum_{i\in S_{jt}} \omega_{ijt-1} d\ln \bar{p}^{CN}_{-it}$$

[18] This instrumental setup is consistent with renewed interest in the sources of identification of Bartik instruments as in Adao et al. (2018); Borusyak et al. (2018); Goldsmith-Pinkham et al. (2018). In all cases, the instrument is the same (exploiting the inner product of vectors), but the sources of identification are different. Identification comes from exogeneity in either the shifters Bartik (1991); Borusyak et al. (2018), or shares Goldsmith-Pinkham et al. (2018). Given our setup, we follow the latter, and obtain identification under the assumption that lagged input shares are exogenous. We also perform the additional tests proposed in Goldsmith-Pinkham et al. (2018) in Appendix D to support our argumentation for these instruments.

[19] The leave-out-mean also serves as a correction for small-sample bias of the estimator that is otherwise induced including average growth rates (Goldsmith-Pinkham et al. (2018)).

and

$$d\ln P_{jt}^{IV3} = \sum_{i\in S_{jt}} \omega_{ijt-1} d\ln \bar{p}_{-it}^{PC}$$

This approach merits some discussion. First, in our setting, the identifying assumptions are expressed in terms of lagged input shares ω_{ijt-1}, and so the input shares are effectively used as instruments. The exclusion restriction is $\mathbb{E}(\sum_{i\in S_j} \omega_{ijt-1} d\ln \bar{p}_{-it}\varepsilon_{jt}) = 0$, which collapses to $\mathbb{E}(\omega_{ijt-1}\varepsilon_{jt}) = 0$ for all i, under the natural asymptotic assumption of a fixed number of time periods, and buyers and suppliers growing to infinity (Goldsmith-Pinkham et al. (2018)). The intuition is that variation in outcomes is at the j level, and the only component that varies at the j level is ω_{ijt-1}, such that j's have different exposures to i through ω_{ijt-1}. Second, given the identifying assumptions, it's important to observe the true (unbiased) input shares, which follow directly from Shephard's lemma. Third, all five proposed instruments rely on firm-specific input shares, which are highly heterogeneous across firms. This is equivalent to a large sparse matrix which exploits exogenous variation along these dimensions. Otherwise stated, identification rests on the exogenous and heterogeneous exposure to average price shocks, with non-completely overlapping sets (non-transitivity) of peers (e.g. Bramoulle et al. (2009)). Moreover, the directed acyclical graph structure of the data allows approach causality and identification as in Pearl (2000).

Third, for competitors' prices, we use a similar strategy as the Bartik (1991) approach just described. However, it is hard to maintain that market shares λ_{ljt-1} would be exogenous and thus we cannot instrument the output prices of competitors directly. Therefore, we instrument the cost shocks of competitors using a shift-share design. Again, we construct two instruments, one for domestic inputs to competitors, and one for imported inputs. In particular,

$$d\ln \mathscr{P}_{-jt}^{IV4} = \sum_{l\neq j\in PC4} \lambda_{ljt-1} \left(\sum_{m\neq i\in S_{lt}} \omega_{mlt-1} d\ln \bar{p}_{-mt}^{CN} \right)$$

and

$$d\ln \mathscr{P}_{-jt}^{IV5} = \sum_{l\neq j\in PC4} \lambda_{ljt-1} \left(\sum_{m\neq i\in S_{lt}} \omega_{mlt-1} d\ln \bar{p}_{-mt}^{PC} \right)$$

where m are other suppliers to competitor l who are not also supplying j, with a weighted average leave-out-mean across all m within the same product category. The exclusion restriction is then $\mathbb{E}\left(\omega_{mlt-1}\varepsilon_{jt}\right) = 0$ for all m. Again, we exploit the observed nature of the network to reinforce the exclusion restriction assumption. In particular, we drop competitors l that are also suppliers to j, and intra-firm transactions to obtain a directed acyclical graph.

4 Results on price updating

Table 1 reports our baseline results from estimating equation eq(8). Columns (i)-(iii) report the coefficients by estimation of eq(8) using OLS. Columns (i)-(iii) shows results with year fixed effects, year and sector fixed effects, and sector-year fixed effects respectively. Sectors are defined as aggregated NACE industries.[20] All results include robust clustered standard errors at this sector level.

Estimated coefficients are very stable across fixed effects specifications. The estimated cost pass-through coefficient is .26, indicating that a 1% shock to input prices by suppliers of j, $d\ln P_{jt}$, correlates with a .26% increase in the output price of j on average, all else equal. The coefficient on the productivity shock $d\ln z_{jt}$ is -.11, implying that an increase in productivity correlates with a downward adjustment of output prices. Finally, price changes of other firms, $d\ln \mathscr{P}_{-jt}$, are also important: on average, and conditional on cost shocks, a 1% increase in the price of producers

[20] In particular, we group NACE Rev2 10-12, 13-18, 19-25 and 26-33 respectively, to obtain five "sectors".

in the same the 4-digit product PC product category, relates to a .35% increase in j's own price on average, entirely accruing to an increase in its markup.

These OLS estimates are likely to suffer from endogeneity bias however. We re-estimate eq(8) deploying an IV approach with the constructed instruments. For these instruments to be valid, they have to be correlated with the endogenous variables (relevance) and only affect the dependent variable through these endogenous variables (exclusion). Table 2 reports the results of the first stage regressions for the three different specifications of the IV estimation, using GMM. Each IV estimation involves two first stage regressions, one for each endogenous variable. In all cases, the first stage F-test is very high (larger than 190 in each specification) with a corresponding p-value of .00, strongly supporting the relevance of the instruments used. Conditional on $d \ln z_{jt}$ and ξ, there is a strong correlation between the endogenous variables and their proposed instruments, both in terms of significance and size.[21]

Next, columns (iv)-(vi) in Table 1 report the IV regression results using GMM. A Hansen J-test cannot reject the null of the the overidentifying exclusion restrictions at the 1% level in all specifications. The estimated IV coefficients confirm the bias in the cost pass-through coefficient from using OLS, and the estimated coefficient has now increased to .52. From eq(6), the implied elasticity of a change in output price to markups is $\frac{\partial \ln \mu_{jt}}{\partial \ln p_{jt}} = 1 - 1/\beta = -0.92$; i.e. on average, producers j face elastic changes in markups: a 1% increase in output price relates to a 0.92% decrease in markups. The coefficients on productivity shocks and other firms' price changes remain largely unchanged. Again, from eq(6), the structural interpretation of the productivity shock elasticity on output is $\frac{\partial \ln y_{jt}}{\partial \ln z_{jt}} = \frac{\gamma}{\beta} = .21$. Similarly, the response in j's markup from changes in other prices is structurally recovered from $\frac{\partial \ln \mu_{jt}}{\partial \ln \mathscr{P}_{-jt}} = \frac{\delta}{\beta} = .71$. When accounting for year and sector fixed effects, all estimated coefficients are largely unaffected. This suggests that business cycles or sector-level characteristics do not fundamentally explain j's average cost pass-through, nor its average response to price changes in its environment.

The results extend earlier results on incomplete pass-through for importers or particular sectors with structural assumptions. Taking into account the network structure of production, and estimating the pricing equation for both domestic and importing firms in Belgium, these results confirm that empirically, models of price setting behavior with constant markups, such as perfect competition or monopolistic competition with CES preferences are refuted, at least at the average.

We perform several robustness tests. Tables are relegated to Appendix D, and we briefly discuss the results here.

First, we perform additional tests to support the argumentation of the Bartik instruments, as suggested by Goldsmith-Pinkham et al. (2018). In particular, we re-estimate eq(8) using 2SLS and LIML as alternative estimators. Since their underlying assumptions are different, these estimators might not give the same results. Under the null of constant effects, alternative estimators should deliver similar point estimates, and potential diversion of these point estimates should be reason to worry. Additionally, overidentification tests for these alternative estimators provide more formal tests for potential model mis-specification. All results are virtually identical to the baseline GMM results, and all regressions pass the different overidentification tests. This reinforces our argumentation of using the proposed set of instruments.

Second, the main remaining threat to our identification assumptions is that demand shocks might be correlated between j and domestic or international product prices. We also estimate eq(8) on subsets of the instruments, as in Duranton & Turner (2012); Amiti et al. (2016). The intuition is that different instruments exploit different sources of variation, in this case domestic versus international products, such that potential sources of endogeneity are also different across these instruments. Again, results using only domestic or only international instruments are very similar, and pass the overidentification tests in every case.

[21] A Cragg and Donald test strongly rejects the null of weak instruments at any plausible level of significance. An additional endogeneity test using Hayashi's C statistic strongly reject the null hypothesis that the endogenous variables can be treated as exogenous.

Table 1: Price updating regressions.

	OLS			IV		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$
$d\ln P_{jt}$	0.260*	0.259*	0.256*	0.521***	0.524***	0.531***
	(0.065)	(0.065)	(0.064)	(0.063)	(0.063)	(0.062)
$d\ln z_{jt}$	-0.106*	-0.109**	-0.109**	-0.107***	-0.110***	-0.109***
	(0.023)	(0.023)	(0.023)	(0.005)	(0.005)	(0.005)
$d\ln \mathscr{P}_{-jt}$	0.362**	0.347**	0.345**	0.377***	0.368***	0.403***
	(0.051)	(0.047)	(0.046)	(0.090)	(0.090)	(0.098)
FE	year	year + sector	year×sector	year	year + sector	year×sector
N	33,787	33,787	33,787	33,718	33,718	33,718
J-test χ^2				3.70	3.21	4.72
[p-value]				[.30]	[.36]	[.19]

Note: Columns (i)-(iii) report OLS estimates, columns (iv)-(vi) reports the second stage of IV estimates employing GMM with 5 instruments. All regressions are pooled over the years 2004-2014. The IV specifications pass all validity tests. Hansen's over-identification J-test statistic cannot reject the null hypothesis that the over-identifying restrictions are valid at the 1% level. Robust standard errors, clustered at the aggregated sector level (5 clusters) in parentheses. Significance: * < 5%, ** < 1%, *** <0.1%.

Table 2: First stage results, instrumental variables estimation.

	Year fixed effects		Year + sector fixed effects		Year×sector fixed effects	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d\ln P_{jt}$	$d\ln \mathscr{P}_{-jt}$	$d\ln P_{jt}$	$d\ln \mathscr{P}_{-jt}$	$d\ln P_{jt}$	$d\ln \mathscr{P}_{-jt}$
$d\ln z_{jt}$	.010***	-.000	.010***	-.002**	.009***	-.002**
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
$d\ln P^{TFP}_{jt}$	-.084***	.027**	-.087***	.018*	-.087***	.015
	(.020)	(.008)	(.020)	(.008)	(.020)	(.008)
$d\ln P^{PC}_{jt}$	.673***	.220***	.671***	.207***	.653***	.165**
	(.068)	(.058)	(.068)	(.058)	(.068)	(.061)
$d\ln P^{CN}_{jt}$	.831***	.179***	.832***	.181***	.824***	.159***
	(.019)	(.014)	(.019)	(.014)	(.019)	(.014)
$d\ln \mathscr{P}^{PC}_{-jt}$	1.123***	1.605***	1.123***	1.513***	.762***	.727***
	(.171)	(.144)	(.171)	(.144)	(.188)	(.166)
$d\ln \mathscr{P}^{CN}_{-jt}$	.131***	.798***	.133***	.801***	.121***	.768***
	(.019)	(.032)	(.019)	(.031)	(.020)	(.032)
First stage F-test	585	334	583	332	493	190
[p-value]	[.00]	[.00]	[.00]	[.00]	[.00]	[.00]

Note: Regression results for the first stages of the IV estimation. For each specification, there are two first stages, one for each endogenous and instrumented variable. Columns (i)-(ii) refer to the first stages of column (iv) in Table 1, etc. Robust standard errors, clustered at the aggregated sector level (5 clusters) in parentheses. Significance: * < 5%, ** < 1%, *** <0.1%.

5 Propagation and aggregation

5.1 Effects of a shock on producer output prices

Propagation: First, eq(5) can be re-written in reduced form. The total impact of an exogenous shock to each producer's output price at time t is then

$$d\ln\mathbf{p} = [I - \beta\Omega - \delta\Lambda]'^{-1}(\gamma d\ln\mathbf{z} + \eta d\xi) + \tilde{\varepsilon} \tag{9}$$

where $d\ln\mathbf{p}$ is the vector of output prices for any producer j, β is the $N\times N$ diagonal matrix of pass-through rates, Ω is the $N\times N$ matrix of cost-based expenditure weights ω_{ijt}, δ is the $N\times N$ diagonal matrix of reaction to other prices, Λ is the $N\times N$ block-diagonal matrix of weights λ_{lj} for the environment's price index. Candidates for exogenous shocks are then productivity shocks $d\ln\mathbf{z}$ with allocated scalar of elasticities γ, and demand shocks $d\xi$ with appropriate η. Finally, $\tilde{\varepsilon} \equiv [I - \beta\Omega - \delta\Lambda]'^{-1}\varepsilon$ is a transformation of the initial error term.[22] Intuitively, exogenous shocks accumulate through the firm-level production network Ω, ultimately affecting producer j's output price $d\ln p_j$. At the same time however, incomplete pass-through dampens the propagation effect, while responses to the environment price indices attenuate the resulting price change.

Aggregation: Second, a change in the producer price index due to a shock can be written as

$$d\ln\mathbb{P} = \sum_j \nu_j d\ln p_j(\beta,\Omega,\delta,\Lambda; d\ln\mathbf{z}, d\xi) \tag{10}$$

with ν_j being an appropriate (fixed) weight depending on the particular price index envisioned. A change in the index $\mathbb{P}$ is ultimately a function of (i) heterogeneity in the network structure of production, (ii) the amount of incomplete pass-through of cost shocks through the production network, (iii) the response to environment prices, (iv) the nature of the shock, and (v) the weights of the index.

Eq(9) nests several implied pricing equations, and it is useful to revisit some of these. Under no or constant markups, pass-through is complete, and eq(9) collapses to $d\ln\mathbf{p} = \mathbf{A}(d\ln\mathbf{z} + d\xi)$, where $\mathbf{A} = [I-\Omega]'^{-1} = \sum_{k=0}^{\infty}\Omega^k$ denotes the Leontief inverse with cost shares ω_{ij} (see also Carvalho & Tahbaz-Salehi (ming)). It is important to underline that even this simplified specification accommodates various models of imperfect competition, as the relevant object is the vector of changes in prices, not the level of output as is the typical application of the Leontief inverse. Exogenous shocks are then ultimately fully passed on to output prices of all producers j, with the total exposure to the shock given by the cumulative sum of direct and indirect input weights $a_{ij} \in \mathbf{A} = \omega_{ij} + \sum_{m=1}^{n}\omega_{im}\omega_{mj} + \dots$.

This pricing equation is a generalization of the shares argument for pass-through in Burstein & Gopinath (2014): with complete pass-through, the effect of a shock to import prices on output prices is simply scaled by the share of those imports in input expenditures of the importer. Importantly however, local costs are not constant anymore due to potential linkages between importers and domestic firms, and the propagation of these shocks amplifies the total impact of imported shocks. It is also a restatement of the canonical model in the networks shock propagation literature: with complete pass-through, any shock to input prices or productivity is ultimately fully borne by the final consumer, appropriately weighted by the particularities of the production network, where weights are given by e.g. Domar weights in Hulten (1978); Gabaix (2011), the influence vector of Acemoglu et al. (2012), or more intricate centralities as in Baqaee (2018).

[22] The matrix Λ is block-diagonal as it has entries > 0 for producers in the same environment as j and 0 otherwise. Ω and Λ are both row-stochastic. Existence of equilibrium is ensured when $||\beta_j\omega_{ij} + \delta_j\lambda_{lj}|| \leq 1$ for all j, as $[I - \beta\Omega - \delta\Lambda]$ is non-singular. Note also that ε can be serially correlated through the network structure. This is explicitly dealt with in the GMM estimation procedure.

5.2 Application: exchange rate pass-through and exchange rate disconnect

There is a close relationship between our framework and the canonical model of exchange rate pass-through in international macro as in e.g. Burstein & Gopinath (2014) or Amiti et al. (2016). In particular, the change in a firm j's output price due to a shock in the real exchange rate to imported inputs, or the exchange rate pass-through of imports into domestic prices for importers, is simply given by an application of eq(2):[23]

$$\frac{d\ln p_{jt}}{d\ln e_t} = \tilde{\beta}_{jt} \sum_{i\in S_{jt}} \omega_{ijt-1} \frac{d\ln p_{it}}{d\ln e_t} + \tilde{\delta}_{jt} \frac{d\ln \mathscr{P}_{-jt}}{d\ln e_t} \tag{11}$$

where e_t is the relevant real exchange rate for imports in other currencies to firm j and $\tilde{\beta}$ and $\tilde{\delta}$ represent the elasticities with respect to exchange rates. For expositional simplicity, we assume that exchange rate shocks are orthogonal to (i) productivity shocks and (ii) the idiosyncratic demand shifters.

Eq(11) is reminiscent of a standard exchange rate pass-through regression, with a few notable differences. First, we linearly separate changes in marginal costs into individual input price changes and productivity shocks. This implies we can separately shock individual imports, by input and by individual exchange rate, rather than the need to shock the marginal cost as a whole. This presents a clear structural interpretation of an exchange rate shock going through the channel of prices. The contribution of an exchange rate shock (holding constant domestic input prices and productivity) is thus simply weighted by the joint input share of imports in that particular currency. Second, and more importantly, the exchange rate shock can have an effect not only on importers, but also on domestic firms who are (in)directly sourcing from importers.

6 Conclusion

Firms can change their prices as a function of both changes in their cost structure (either through input prices or productivity shocks) and changes in other prices in their environment. The non-parametric pricing model is very general, and can be applied to various settings of market structure and demand. The particularities of each parameterization will impose further restrictions on the sign and size of the elasticities that can be taken to the data.

Importantly, full pass-through of cost shocks and constant markups are empirically refuted, at least on average. This includes the rejection of various models, such as the canonical model of international trade with monopolistic competition with CES preferences, thereby shutting down important welfare mechanisms that operate through these channels.

[23] An exchange rate shock on imported inputs is natural in our framework, as in Amiti et al. (2016). By introducing exports and labeling the marginal cost in terms of the destination country, our framework also encompasses exchange rate pass-through on the output side as in Burstein & Gopinath (2014).

Appendices

A Data sources and preparation

The empirical analysis in this paper draws mainly from four micro-level data sources, administered at the National Bank of Belgium (NBB). These include (i) information on production values and quantities at the firm-product level from the Prodcom Survey for Belgium, (ii) international trade data on values and quantities at the firm-product-country level from Intrastat and Extrastat, (iii) domestic supplier-buyer relationship values from the NBB B2B Transactions Dataset, and (iv) firm-level characteristics from the annual accounts and the statistical office at the NBB. Firms are identified by a unique corporate registration number from the Crossroads Bank for Enterprises, which allows for unambiguous merging across the different datasets. We have also developed a custom concordance procedure for product classifications in international trade (CN) and Prodcom (PC), which relies on various correspondence and harmonization tables at Eurostat. Next, we briefly describe the sources, dimensions and cleaning of the different datasets.

Prodcom Survey

The first source for firm-product prices is the Prodcom Survey. This survey collects information on monthly production statistics of manufactured goods and industrial services for firms in the Prodcom Classification (PC), covering the "Mining and Quarrying" and "Manufacturing" sectors in the NACE classification. For Belgium, the survey is organized and collected by Statistics Belgium, and products are defined at the highly disaggregated 8-digit level of the Prodcom Classification (PC8), covering roughly 2,400 products produced in Belgium.[24]

The Prodcom Survey is not a (stratified) random sample, but an exhaustive coverage of firms above certain reporting thresholds: all firms that produce goods covered by the classification, and that have either at least 20 persons employed, or a turnover of at least 3,928,137 euro in the previous year, have to submit a monthly report to Statistics Belgium.[25] Reporting thresholds are defined at the consolidated level, such that daughters of surveyed firms also report their production and sales, independent of their size.

The database reports, for each *firm*, monthly *values* and *quantities* for all of its PC8 *products* produced in Belgium and sold in the reference period. Values are reported in current euros, and quantities are reported in one of several possible *units*. Over two thirds of observation are in kilograms, and other units include liters, meters, square meters, kilowatt, kilograms of active substance etc.[26] It is important to account for these units of measurement when international trade data and Prodcom data are concorded later on.[27] The survey is collected in an electronic format, which includes automatic checks when filing the data and additional checks and cross-references with both micro and macro data by Statistics Belgium, to ensure a very high quality of reporting.

Information for all firms with their main activity in "Mining and Quarrying" or "Manufacturing" is retained for

[24] For example, in 2014, within the 6-digit grouping of "*Polymers of ethylene, in primary forms*" (20.16.10), code 20.16.10.35 refers to "*Linear polyethylene having a specific gravity < 0,94, in primary forms*", code 20.16.10.39 is *"Polyethylene having a specific gravity < 0,94, in primary forms (excluding linear)*", code 20.16.10.50 refers to *"Polyethylene having a specific gravity of ≥ 0,94, in primary forms*", etc.

[25] See the National Reference Metadata for Belgium at Eurostat for more info.

[26] For example, clothing is measured per unit or per pair. Wood is in cubic meters or kilograms. Production of chemicals is mostly reported in kilograms of main chemical ingredient or in kilograms. Pesticides and related are reported in kilogram of active substance. Wicks are in kilometers. Strong liquor is in liters of alcoholic content at 100%, while Champagne, wines and beers are reported in liters. Cigarettes are per piece, while tobacco is in kilograms. Finally, some PC codes have to be reported only in values, but not quantities, mostly representing industrial manipulations or services (e.g. bleaching of leather, dying of textile) instead of products.

[27] For example, the Prodcom sales data is corrected for net exports to obtain domestic unit values. To that end, we concord the trade data (CN) with the Prodcom data (PC) and identify products that can be corresponded from CN to PC. When subtracting quantities, we ensure that both products are reported in the same unit to avoid possible mis-measurement of values or quantities.

the period 2002-2014. Values and quantities are aggregated from monthly to yearly observations to match with the other datasets at the yearly level. We then generate firm-product unit values – our proxy for prices – as values over quantities. It is necessary to obtain domestic output prices, both to evaluate domestic competition and as a measure for input prices to other firms. Hence, net exports (i.e. after correction for re-exports of the same product by the same firm) are subtracted from PC sales at the firm-product level, using a custom developed PC-CN concordance method described below. The method accounts for possible one-to-one, one-to-many and many-to-one changes in the classification system, and we ensure that products are reported in the same units when concorded. To avoid induced measurement error, net exports are only corrected for if the resulting domestic price is not greater than 10 times or smaller than 1/10 the median price for that PC8 product in that year.

Finally, changes in domestic firm-product output prices, $d\ln p_{jt}$, are calculated as log differences in unit values from year $t-1$ to t. To deal with changes in the PC classification between any two years, we exploit our custom concordance method again. Finally, we trim log-differenced unit values at ± 1. The main results in the paper are very similar if we trim or winsorize at ± 0.5, or trim the bottom and top 5^{th} percentiles of the distribution instead.

International trade data

The second source for firm-product prices is on imports and exports of goods from the Intrastat (intra-EU) and Extrastat (extra-EU) declarations for Belgium. Goods are reported at the 8-digit level of the Combined Nomenclature (CN8) classification system, a further refinement of the international 6-digit Harmonized System classification system, and covers around 10,000 products in a given year.

Trade of EU firms from and to non-EU countries (Extrastat) are recorded by customs authorities, and forwarded to the NBB for trade relating to Belgian firms. Reporting thresholds are at 1,000 euro, above which firms have to report all of their trade by product-country. Trade between EU firms (Intrastat) is labeled arrival and dispatch of goods, and firms send a monthly electronic report directly to the NBB, replacing traditional customs reporting. The reporting threshold for Intrastat is higher, and has changed over our sample period, from 250,000 euro to 700,000 euro for arrivals, and to 1 million euro for dispatches.

Observations in the resulting trade dataset are at the firm-product-country-year level spanning the years 2002 to 2014. Similarly to the Prodcom data structure, the database reports for each *firm*, monthly *values* and *quantities* for all of its CN8 *products* imported or exported, but now at the *country* of origin or destination level. Values are reported in current euros, and quantities are reported in one of several *units*. At the CN8 level, most quantities are recorded in kilograms. Depending on the particular product, some quantities are also recorded in a secondary unit (which is the PC8 unit if there exists a concordance between the CN8 and PC8 products).

Data cleaning and preparation is similar to the Prodcom method above. We briefly discuss some elements particular to the CN data preparation. First, prices (unit values) are now calculated at the firm-product-county-year level. Hence, the same CN8 product imported from or exported to different countries might face different prices (pricing to market). Second, imports can be used as inputs in production, and we only consider imports that contribute to the marginal cost of the firm when constructing changes in the firm-level input price index. Therefore, (i) the value of imports that are re-exported without further manipulation and (ii) capital goods inputs, are dropped from the variable cost input bundle. Capital goods are identified from the concordance tables between CN and BEC (Broad Economic Classification) at Eurostat, and imports that are marked as capital goods (BEC codes 410 and 521) are dropped from the import bundle. Third, in order to obtain domestic prices (i.e. corrected for net exports) in the PC data, export values and quantities for each firm-year are aggregated across destinations, and domestic prices for product k by firm j are then obtained as $p_{jkt} = \frac{\text{total value - net export value}}{\text{total quantity - net export quantity}}$. Resulting domestic prices greater than 10 times or smaller than 1/10 the median price for that CN8 product-year are not corrected, to avoid induced measurement error.

Finally, changes in prices for net imported products are obtained at the firm-product-country level as log differences in unit values. We use the same methodology for changes in CN8 from year to year, and for one-to-one, one-to-many and many-to-one correspondences as the procedure for PC8, scrutinizing the unit of measurement. Log-differenced unit values are trimmed at ± 1. Alternative thresholds for trimming or winsorizing have no significant bearing on our results.

NBB B2B Transactions Dataset

Next, exhaustive information on firm-level supplier-buyer relationships across all economic activities in Belgium is obtained from the NBB B2B Transactions Dataset (Dhyne et al. (2015)). This dataset permits to map the whole domestic production network. Together with the import trade data, it allows to construct the full decomposition of input expenditures for every firm j by supplier i.

The dataset contains the values of yearly sales relationships among all VAT-liable Belgian enterprises for the years 2002 to 2014, and is based on the VAT listings collected by the tax authorities. At the end of every calendar year, all VAT-liable firms have to file a complete listing of their Belgian VAT-liable customers over that year.[28] An observation in this dataset refers to the sales value in euro of firm i selling to firm j within Belgium, net of the VAT amount due on these sales. The reported value is the sum of invoices from i to j in a given calendar year. Whenever this aggregated value is 250 euro or more, the relationship has to be reported. Fines are imposed for late and/or erroneous reporting. Each observation m_{ij} is directed, as firm i can be selling to j, but not necessarily the other way around, i.e. $m_{ij} \neq m_{ji}$.

We retain producers j that are sellers in the Prodcom data, and keep all input suppliers i across all economic activities to these firms. We drop suppliers i to j that mainly produce capital goods (NACE Rev.2 Divisions 28 and 41-43), as these goods are considered not to be part of the marginal cost bundle of their customers. We then calculate the total expenditures on variable costs on domestic suppliers.

Importantly, we also exploit the unique dimensions of the network data to attenuate potential simultaneity bias in the pass-through regressions. In particular, we flag (i) relationships of firms j that are also supplying their suppliers i (cyclicality), and (ii) suppliers i to j that are in the same PC 4-digit category as j (competitors). For these flagged observations, we do not include their price changes, but keep the input expenditures as to not bias the true input shares. Finally, we also account for intra-firm trade by dropping relationships between any two VAT identities that belong to the same group, as these transactions might not reflect market prices (see Tintelnot et al. (2017) for the applied methodology).[29]

Annual accounts

Finally, typical firm characteristics are obtained from the annual accounts at the Central Balance Sheet Office at the NBB. We extract data for the years 2002 to 2014, which are mainly used to estimate a quantity based productivity measure, *TFPq*.

All firms with economic activities in Belgium have to submit annual accounts after closure of their fiscal years.[30] We extract information on *capital* (stock of fixed assets including material and financial fixed assets in euro, codes 20-28 in the accounts), *labor cost* (total cost of wages, social securities and pensions in euro, code 62), and *employment* (average number of full-time equivalent (FTE) employees, code 9087). We annualize the flow variables from fiscal years to calendar years by pro-rating on a monthly base. This ensures that the variables are consistent with the calendar

[28] Sample VAT listings forms can be found at here (French) and here (Dutch).

[29] For example, in 2014 cyclicality accounts for around 951,000 transactions out of 17 million domestic relationships, while intra-firm trade amounts to around 17,000 relationships.

[30] See here for filing requirements and exceptions. See here for the size criteria and filing requirements for either full-format or abridged annual accounts.

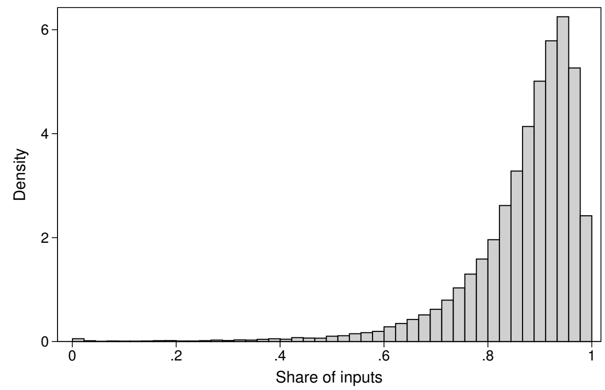


Figure 3: Share of continuing inputs.

year observations in the NBB B2B Transactions Dataset.

Finally, we extract the main economic activity (defined as the sector with the highest revenue share for multi-product firms) of each firm at the NACE 4-digit level from the Crossroads Bank of Enterprises.[31]

B Additional descriptive statistics

This section provides additional statistics on the stylized facts presented in the main text. Table 3 shows the distribution of output price changes at the firm-product level by 2-digit NACE sector for the year 2014. Sectors with at least 10 observations are reported. Table 4 shows the distribution of output price changes by year, using the main output for each firm instead of all products in the main text. Similarly, Table 5 shows the distribution using a sales-weighted average of product price changes at the firm level.

Figure 3 shows the full distribution of the share of input expenditures on suppliers that are continuing from $t-1$ to t, pooled across all firms and years. On average, 90% of the value of the input bundle at time t consists of suppliers that were also supplying to j at $t-1$.

C Model with multi-product firms

This section develops an extended version of the model, in which firms potentially produce multiple outputs. Two variants of the model are presented, in which the unit of observation is either the firm-product or the firm as a whole. This distinction has implications for the identification assumptions when taken to the data.

[31] For some of the analysis, we use harmonized NACE Rev.2 codes. This ensures that firms can be consistently grouped into sectors over time, for the production function estimation and the use of sector fixed effects in the pooled estimations. For cross-sectional analysis, non-harmonized NACE codes are used, to ensure a clean mapping between PC and NACE, as the first 4 digits of the PC codes coincide with the 4-digit NACE codes.

Table 3: Distribution of producer price changes, by sector (2014).

					Percentiles			
NACE Rev.2 Division	N	mean	sd	p5	p25	p50	p75	p95
10 – Manufacture of food products	1,547	-.01	0.18	-0.27	-0.07	0.00	0.04	0.25
11 – Manufacture of Beverages	84	.011	0.20	-0.25	-0.05	0.00	0.06	0.37
13 – Manufacture of textiles	169	0	0.23	-0.49	-0.05	0.00	0.08	0.32
14 – Manufacture of wearing apparel	34	-.007	0.19	-0.41	-0.07	0.00	0.05	0.27
16 – Manufacture of wood [...]	174	-.008	0.19	-0.32	-0.02	0.01	0.06	0.21
17 – Manufacture of paper and paper products	127	-.009	0.21	-0.44	-0.03	0.00	0.05	0.27
18 – Printing and reproduction of recorded media	37	-.054	0.21	-0.70	-0.02	0.00	0.02	0.09
19 – Manufacture of coke and refined petroleum products	27	-.041	0.21	-0.69	-0.03	0.00	0.02	0.17
20 – Manufacture of chemicals and chemical products	571	.002	0.22	-0.31	-0.07	0.00	0.07	0.35
21 – Manufacture of basic pharmaceutical products[...]	29	.009	0.20	-0.32	-0.07	0.00	0.10	0.30
22 – Manufacture of rubber and plastic products	225	-.006	0.23	-0.40	-0.06	0.00	0.04	0.33
23 – Manufacture of other non-metallic mineral[...]	315	.014	0.15	-0.15	-0.02	0.00	0.03	0.22
24 – Manufacture of basic metals	124	-.02	0.21	-0.34	-0.08	-0.01	0.04	0.26
25 – Manufacture of fabricated metal products[...]	436	.012	0.22	-0.35	-0.05	0.00	0.04	0.51
26 – Manufacture of computer, electronic and optical products	61	.02	0.28	-0.44	-0.08	0.01	0.11	0.48
27 – Manufacture of electrical equipment	76	.037	0.21	-0.23	-0.05	0.00	0.09	0.42
28 – Manufacture of machinery etc.	157	.043	0.25	-0.37	-0.04	0.01	0.13	0.48
29 – Manufacture of motor vehicles, trailers and semi-trailers	33	-.019	0.19	-0.52	-0.06	-0.01	0.05	0.29
30 – Manufacture of other transport equipment	15	.046	0.25	-0.52	-0.04	0.04	0.15	0.57
31 – Manufacture of furniture	212	.01	0.17	-0.28	-0.03	0.00	0.05	0.27
32 – Other manufacturing	16	-.03	0.28	-0.81	-0.13	0.01	0.12	0.48
33 – Repair and installation of machinery and equipment	11	.096	0.22	-0.05	0.00	0.02	0.09	0.72

Table 4: Distribution of producer price changes, main output (2003-2014).

					Percentiles			
Year	N	mean	sd	p5	p25	p50	p75	p95
2003	3,749	.004	0.21	-0.33	-0.05	0.00	0.05	0.37
2004	3,676	.012	0.20	-0.30	-0.03	0.00	0.06	0.34
2005	3,428	.011	0.20	-0.29	-0.04	0.00	0.08	0.31
2006	3,326	.014	0.19	-0.27	-0.03	0.00	0.07	0.31
2007	3,468	.03	0.19	-0.24	-0.02	0.02	0.09	0.32
2008	2,505	.032	0.24	-0.34	-0.03	0.02	0.10	0.43
2009	2,623	-.014	0.20	-0.34	-0.07	0.00	0.04	0.28
2010	2,451	.002	0.21	-0.33	-0.05	0.00	0.06	0.30
2011	2,305	.029	0.19	-0.23	-0.02	0.01	0.09	0.33
2012	2,166	.013	0.21	-0.31	-0.03	0.01	0.07	0.32
2013	2,090	-.005	0.19	-0.32	-0.04	0.00	0.05	0.25
2014	2,026	0	0.19	-0.30	-0.05	0.00	0.05	0.30
All	33,813	.011	0.20	-0.30	-0.04	0.00	0.07	0.33

Table 5: Distribution of producer price changes, firm-level weighted average (2003-2014).

					Percentiles			
Year	N	mean	sd	p5	p25	p50	p75	p95
2003	3,749	.004	0.18	-0.28	-0.04	0.00	0.05	0.31
2004	3,676	.01	0.18	-0.24	-0.03	0.00	0.05	0.28
2005	3,428	.01	0.17	-0.24	-0.03	0.00	0.06	0.25
2006	3,326	.015	0.17	-0.21	-0.02	0.00	0.06	0.26
2007	3,468	.028	0.16	-0.19	-0.02	0.01	0.08	0.26
2008	2,505	.031	0.21	-0.28	-0.02	0.02	0.09	0.39
2009	2,623	-.015	0.17	-0.30	-0.06	0.00	0.04	0.23
2010	2,451	.002	0.17	-0.26	-0.04	0.00	0.05	0.25
2011	2,305	.028	0.16	-0.19	-0.01	0.01	0.08	0.28
2012	2,166	.013	0.17	-0.24	-0.02	0.01	0.06	0.28
2013	2,090	-.004	0.17	-0.29	-0.03	0.00	0.04	0.22
2014	2,026	-.002	0.16	-0.23	-0.04	0.00	0.04	0.22
All	33,813	.011	.17	-.25	-.03	.00	.06	.27

C.1 Model at the firm-product level

The cost function of output k by firm j is now

$$y_{jkt}c_{jkt}\left(\left(1+\tau_{1j}\right)p_{1t},...,\left(1+\tau_{nj}\right)p_{nt},z_{jt}\right)+F_{jkt}$$

For our purpose of identifying the sources of output price changes, allowing for multi-product firms requires the following additional assumptions:

Assumption 1. *No physical synergies across products within producers.*

This implies that the cost function of one product is independent of the production of the other products within the firm.

Assumption 2. *Proportionality of inputs to outputs.*

The share of input i allocated to product k by producer j, $\varphi_{ijkt}=\frac{S_{jkt}}{\sum_k S_{jkt}}$ is constant across all inputs i, whereS_{jkt} is the revenue share of k in revenue of j. This assumption implies that all inputs are proportionally allocated to outputs, with their shares corresponding to the respective revenue shares in production. In the data, there is no information on how firms allocate their inputs to producing multiple outputs, and an assumption on this allocation has to be made. This issue is omni-present in firm-level datasets with multi-product firms, and is not specific to our setup. Note that in our setup we have to allocate any domestic PC8 and foreign CN8 inputs to specific output, not only factors of production as in De Loecker et al. (2016).

Under these assumptions, the pricing equation eq(1) can be written at the firm-product level as

$$\ln p_{jkt}=\ln c_{jkt}\left(\left(1+\tau_{1j}\right)p_{1t},...,\left(1+\tau_{nj}\right)p_{nt},z_{jt}\right)+\ln\mu_{jkt}\left(p_{jkt},\mathscr{P}_{-jkt};\xi_{jkt}\right)$$

The fact that productivity is firm-specific (and not firm-product specific) is not necessarily required in our setting, but it follows the large literature on estimating productivity in multi-product firms (see e.g. Bernard et al. (2011); De Loecker et al. (2016)).

Log-differentiating the price equation, we have that the log change in price of output k by producer j can be approximated as

Hence,

$$d\ln p_{jkt}=\sum_{i\in\mathscr{S}_{jt}}\frac{\partial\ln c_{jkt}}{\partial\ln p_{it}}d\ln p_{it}+\frac{\partial\ln c_{jkt}}{\partial\ln z_{jt}}d\ln z_{jt}+\frac{\partial\ln\mu_{jkt}}{\partial\ln p_{jkt}}d\ln p_{jkt}+\frac{\partial\ln\mu_{jkt}}{\partial\ln\mathscr{P}_{-jkt}}d\ln\mathscr{P}_{-jkt}+\frac{\partial\ln\mu_{jkt}}{\partial\xi_{jkt}}d\xi_{jkt}$$

Then,

$$d\ln p_{jkt}=\frac{1}{1-\frac{\partial\ln\mu_{jkt}}{\partial\ln p_{jkt}}}\left(\sum_{i\in\mathscr{S}_{jt}}\omega_{ijt}d\ln p_{it}+\frac{\partial\ln c_{jkt}}{\partial\ln z_{jt}}d\ln z_{jt}+\frac{\partial\ln\mu_{jkt}}{\partial\ln\mathscr{P}_{-jkt}}d\ln\mathscr{P}_{-jkt}+\frac{\partial\ln\mu_{jkt}}{\partial\xi_{jkt}}d\xi_{jkt}\right)\tag{12}$$

where we have exploited the proportionality assumption, $\varphi_{ijkt}=\varphi_{jkt}$ for all i, so that

$$\omega_{ijkt}\equiv\frac{\varphi_{ijkt}x_{ijt}p_{it}(1+\tau_{ij})}{\sum_{i\in\mathscr{S}_{jt}}\varphi_{ijkt}x_{ijt}p_{it}(1+\tau_{ij})}=\frac{x_{ijt}p_{it}(1+\tau_{ij})}{\sum_{i\in\mathscr{S}_{jt}}x_{ijt}p_{it}(1+\tau_{ij})}\equiv\omega_{ijt}$$

Under the above conditions, eq(12) specifies a pass-through regression for multi-product firms, where the change in output price for a particular product k of producer j is evaluated against a combination of input price shocks, productivity shocks and the price index $\mathscr{P}_{-jkt}$.

C.2 Model at the firm level

It is possible to aggregate firm-product price shocks again to the firm level, with one additional assumption.

Assumption 3. *Markup shocks are the same across products within firms,* $\frac{\partial \ln \mu_{jkt}}{\partial \ln p_{jkt}} = \psi_{jt}$ *for all k.*

In this case, changes in output prices are aggregated to a firm-level output price index $d\ln\tilde{P}_{jt}$, with weights given by the revenue share of each product k, φ_{jkt}, so that $d\ln\tilde{P}_{jt} \equiv \sum_k \varphi_{jkt} d\ln p_{jkt}$.

Eq(2) then becomes

$$d\ln\tilde{P}_{jt} = \sum_k \varphi_{jkt}\left(\sum_{i\in\mathscr{S}_{jt}} \frac{\partial \ln c_{jkt}}{\partial \ln p_{it}} d\ln p_{it} + \frac{\partial \ln c_{jkt}}{\partial \ln z_{jt}} d\ln z_{jt} + \frac{\partial \ln \mu_{jkt}}{\partial \ln p_{jkt}} d\ln p_{jkt} + \frac{\partial \ln \mu_{jkt}}{\partial \ln \mathscr{P}_{-jkt}} d\ln \mathscr{P}_{-jkt} + \frac{\partial \ln \mu_{jkt}}{\partial \xi_{jkt}} d\xi_{jkt}\right)$$

Rearranging:

$$d\ln\tilde{P}_{jt}\,(1-\psi_{jt}) = \sum_k \varphi_{jkt}\left(\sum_{i\in\mathscr{S}_{jt}} \frac{\partial \ln c_{jkt}}{\partial \ln p_{it}} d\ln p_{it} + \frac{\partial \ln c_{jkt}}{\partial \ln z_{jt}} d\ln z_{jt} + \frac{\partial \ln \mu_{jkt}}{\partial \ln \mathscr{P}_{-jkt}} d\ln \mathscr{P}_{-jkt} + \frac{\partial \ln \mu_{jkt}}{\partial \xi_{jkt}} d\xi_{jkt}\right)$$

Finally, the pass-through regression becomes

$$d\ln\tilde{P}_{jt} = \beta_{jt}\sum_{i=1}^{n} \omega_{ijt} d\ln p_{it} + \gamma_{jt}\sum_k \varphi_{jkt}\frac{\partial \ln c_{jkt}}{\partial \ln z_{jt}} d\ln z_{jt} + \delta_{jt}\sum_k \varphi_{jkt}\frac{\partial \ln \mu_{jkt}}{\partial \ln \mathscr{P}_{-jkt}} d\ln \mathscr{P}_{-jkt} + \eta_{jt}\sum_k \varphi_{jkt}\frac{\partial \ln \mu_{jkt}}{\partial \xi_{jkt}} d\xi_{jkt}$$

where

$$\begin{cases} \beta_{jt} = \frac{1}{1-\psi_{jt}} \\ \gamma_{jt} = \frac{1}{1-\psi_{jt}}\frac{\partial \ln c_{jkt}}{\partial \ln z_{jt}} \\ \delta_{jt} = \frac{1}{1-\psi_{jt}}\frac{\partial \ln \mu_{jkt}}{\partial \ln \mathscr{P}_{-jt}} \end{cases}$$

If these particular assumptions do not hold for multi-product firms, we cannot directly interpret the coefficients as pass-through and strategic complementarities, as they would be biased by additional cross-elasticities. Table 6 shows the results for the price updating regressions for the multi-product firm extension, using the output price index. Results are very similar to the main results in the paper.

D Robustness

This section discusses alternative specifications and additional results to the main results in Section 4.

First, we show how exploiting the directed structure of the production network allows us to partially cope with simultaneity bias in eq(8). Table 7 shows the OLS results for the standard cleaned data (i), and the data corrected for cyclicality, intra-firm relationships, and suppliers to j that are also competitors. As expected, OLS coefficients are biased downwards due to simultaneity, and point estimates increase when directly dealing with simultaneity in the data, although modestly.

	OLS			IV		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d\ln\tilde{P}_{jt}$	$d\ln\tilde{P}_{jt}$	$d\ln\tilde{P}_{jt}$	$d\ln\tilde{P}_{jt}$	$d\ln\tilde{P}_{jt}$	$d\ln\tilde{P}_{jt}$
$d\ln P_{jt}$	0.257*	0.256*	0.253*	0.493***	0.496***	0.502***
	(0.063)	(0.063)	(0.062)	(0.059)	(0.059)	(0.059)
$d\ln z_{jt}$	-0.103*	-0.105*	-0.105*	-0.104***	-0.106***	-0.106***
	(0.023)	(0.023)	(0.023)	(0.004)	(0.004)	(0.004)
$d\ln\mathscr{P}_{-jt}$	0.336**	0.321**	0.318**	0.373***	0.363***	0.406***
	(0.049)	(0.046)	(0.047)	(0.085)	(0.086)	(0.093)
FE	year	year + sector	year×sector	year	year + sector	year×sector
N	33,787	33,787	33,787	33,718	33,718	33,718
J-test χ^2				3.99	3.67	4.84
[p-value]				[.26]	[.30]	[.18]

Note: Columns (i)-(iii) report OLS estimates, columns (iv)-(vi) reports the second stage of IV estimates employing GMM. All regressions are pooled over the years 2003-2014. Robust standard errors between brackets, all clustered at the aggregated sector level. Significance: * < 5%, ** < 1%, *** <0.1%.

Table 6: Price updating regression with multi-product firms.

Table 7: Dealing with simultaneity bias (OLS).

	Uncorrected	Corrected
	(i)	(ii)
Dep. var.	$d\ln p_{jt}$	$d\ln p_{jt}$
$d\ln P_{jt}$	0.247*	0.275*
	(0.056)	(0.066)
$d\ln z_{jt}$	-0.091**	-0.091**
	(0.019)	(0.019)
$d\ln\mathscr{P}_{-jt}$	0.370**	0.393**
	(0.052)	(0.063)
N	33,787	33,787

*Note:*Regressions are pooled over the years 2004-2014. Robust standard errors, clustered at the aggregated sector level between brackets. Significance: * < 5%, ** < 1%, *** <0.1%.

Table 8: Alternative IV estimators.

	LIML			2SLS		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dep. var.	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$
$d\ln P_{jt}$	0.521***	0.524***	0.531***	0.521***	0.524***	0.531***
	(0.080)	(0.079)	(0.077)	(0.092)	(0.092)	(0.062)
$d\ln z_{jt}$	-0.107***	-0.110***	-0.109***	-0.107***	-0.110***	-0.109***
	(0.010)	(0.010)	(0.010)	(0.021)	(0.021)	(0.005)
$d\ln \mathscr{P}_{-jt}$	0.373**	0.365***	0.401***	0.374***	0.365***	0.402***
	(0.109)	(0.110)	(0.117)	(0.089)	(0.087)	(0.098)
FE	year	year + sector	year×sector	year	year + sector	year×sector
N	33,718	33,718	33,718	33,718	33,718	33,718
overid test χ^2	4.43	3.89	5.86	4.43	3.89	5.86
[p-value]	[.22]	[.27]	[.12]	[.22]	[.27]	[.12]

Note: Columns (i)-(iii) report LIML estimates, columns (iv)-(vi) reports 2SLS with 5 instruments. All regressions are pooled over the years 2004-2014. For LIML, the overidentification test is the Anderson-Rubin test, while for 2SLS, this is given by the Sargan test. Robust standard errors, clustered at the aggregated sector level (5 clusters) in parentheses. Significance: * < 5%, ** < 1%, *** <0.1%.

Second, we estimate eq(8) again using alternative estimators, and present results in Table 8. Columns (i)-(iii) show results for limited information maximum likelihood (LIML), while columns (iv)-(vi) show results for two-stage least squares (2SLS). Results are virtually identical, suggesting that the different underlying assumptions of the estimators are not inconsistent with each other.

Third, we re-estimate eq(8) using subsets of instruments to evaluate the different sources of variation and potential concerns on endogeneity in different dimensions. All results are very stable across instrument specification and pass the overidentificstion tests. The largest diversion is when the Bartik of international prices for competitors is not included.

Table 9: Subsets of instruments.

	(i)	(ii)	(iii)	(iv)	(v)
Dep. var.	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$	$d\ln p_{jt}$
$d\ln P_{jt}$	0.357**	0.522***	0.408*	0.532***	0.529***
	(0.122)	(0.063)	(0.205)	(0.064)	(0.064)
$d\ln z_{jt}$	-0.106***	-0.108***	-0.107***	-0.107***	-0.107***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)
$d\ln \mathscr{P}_{-jt}$	0.778**	0.353***	0.452**	0.371***	0.362***
	(0.269)	(0.091)	(0.158)	(0.090)	(0.091)
$d\ln P_{jt}^{TFP}$	Yes	Yes	Yes	Yes	
$d\ln P_{jt}^{PC}$	Yes	Yes	Yes		Yes
$d\ln P_{jt}^{CN}$	Yes	Yes		Yes	Yes
$d\ln \mathscr{P}_{-jt}^{PC}$	Yes		Yes	Yes	Yes
$d\ln \mathscr{P}_{-jt}^{CN}$		Yes	Yes	Yes	Yes
N	33,718	33,718	33,718	33,718	33,718
overid test χ^2	1.09	1.05	3.37	2.70	2.85
[p-value]	[.58]	[.59]	[.19]	[.26]	[.24]

Note: Columns (i)-(vii) report GMM estimates with different subsets of instruments. All regressions are pooled over the years 2004-2014 and contain year fixed effects. Robust standard errors, clustered at the aggregated sector level (5 clusters) in parentheses. Significance: * < 5%, ** < 1%, *** <0.1%.

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