

Parallel & Scalable Data Analysis

Introduction to Machine Learning Algorithms

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LECTURE 4

Classification Challenges & Solutions

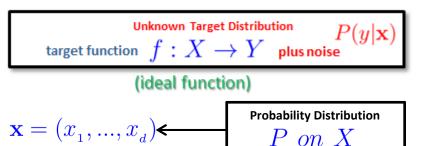
November 24th, 2017 Ghent, Belgium





Review of Lecture 3

- Supervised Classification
 - Finding a target distribution for realistic learning situations
 - Assume unknown probability distribution over the input space
 - Hypothesis search with M models and we pick one



'Probably'

$$\mathcal{H}=\{h\}; \ \ g\in \mathcal{H}$$

Final Hypothesis g pprox f

(set of candidate formulas)

Statistical Learning Theory

 $\Pr\left[\mid E_{in}(g) - E_{out}(g) \mid > \epsilon \mid <= 2Me^{-2\epsilon^2 N} \right]$

'Approximately'

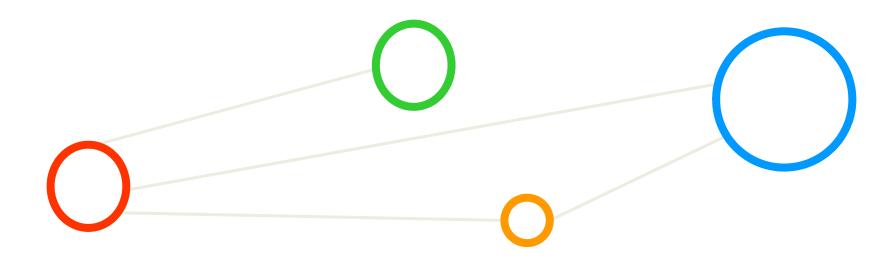
'Number of samples N important for learning'

(set of known algorithms)

$$\Pr \left[\mid E_{in}(g) - E_{out}(g) \mid > \epsilon \right] <= 4m_{\mathcal{H}}(2N)e^{-1/8\epsilon^2 N}$$

Shift the view to the data and we exchange M with growth function that is indeed depending on N

Outline



Outline of the Course

- 1. Machine Learning Fundamentals
- 2. Unsupervised Clustering and Applications
- 3. Supervised Classification and Applications
- 4. Classification Challenges and Solutions
- 5. Regularization and Support Vector Machines
- 6. Validation and Parallelization Benefits

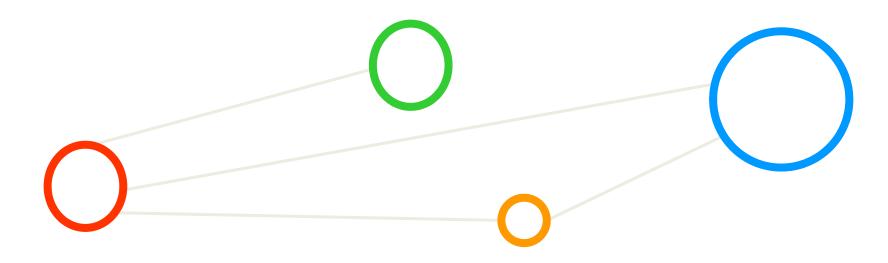


Outline

- Classification Challenges & Solutions
 - Review Practice Experience & Applications
 - Challenge One: Non-Linear Seperable Datasets
 - Challenge Two: Problem of Overfitting
 - Regularization Approach
 - Supervised Classification Models Overview
- Maximal Margin Classifier
 - Term Support Vector Machines Refined
 - Apply Classifier to Datasets
 - Margin as Geometric Interpretation
 - Optimization Problem & Implementation
 - Solving and Limitations of Classifier



Classification Challenges & Solutions



Key Challenges: Why is it not so easy in practice?

Scalability

- Gigabytes, Terabytes, and Petabytes datasets that fit not into memory
- E.g. algorithms become necessary with out-of-core/CPU strategies

High Dimensionality

- Datasets with hundreds or thousand attributes become available
- E.g. bioinformatics with gene expression data with thousand of features

Heterogenous and Complex Data

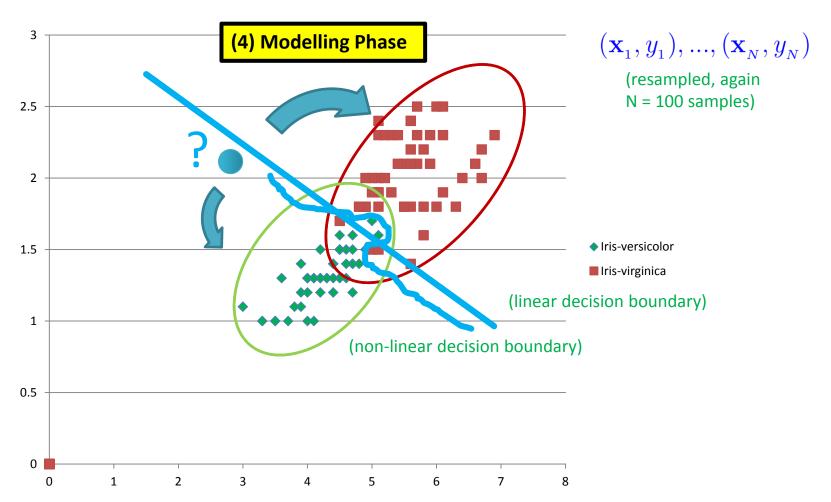
- More complex data objects emerge and unstructured data sets
- E.g. Earth observation time-series data across the globe

Data Ownership and Distribution

- Distributed datasets are common (e.g. security and transfer challenges)
- Key challenges faced when doing traditional data analysis and machine learning are scalability,
 high dimensionality of datasets, heterogenous and complex data, data ownership & distribution

[1] Introduction to Data Mining

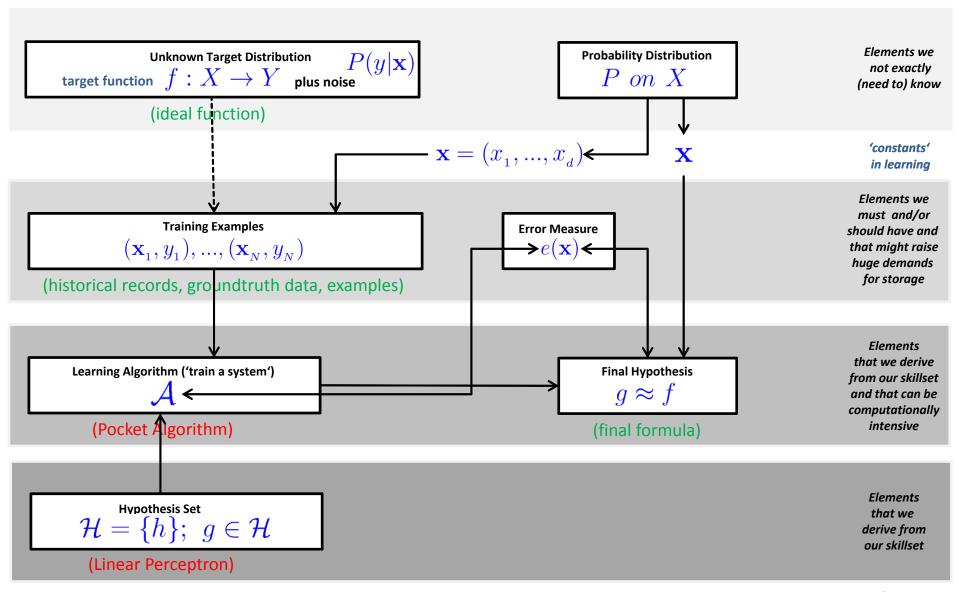
Challenge One - Non-linearly Seperable Data in Practice



(lessons learned from practice: requires soft-thresholds to allow for some errors being overall better for new data)

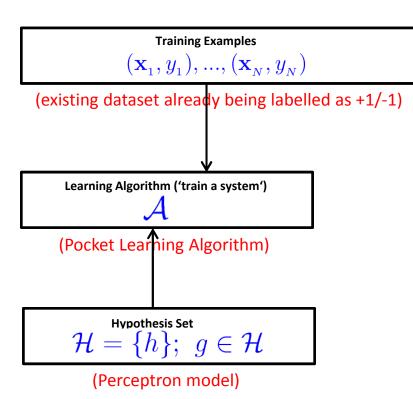
(lessons learned from practice: requires non-linear decision boundaries)

Solution Tools: Linear Perceptron Hypothesis Set & Pocket



Smart Adhancement of PLA – Pocket Algorithm

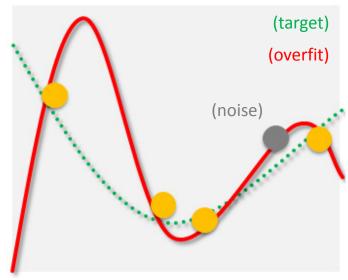
- When: If we believe there is a linear pattern to be detected
 - No assumption: can be non-linearly seperable data



- Basis is still the PLA
- Idea: Put the best solution so far 'in a pocket'
- Best means: Error measure checks per iterations
- Works with non-linearly seperable data
- Needs fixed iterations number (otherwise no convergence of algorithm)

Challenge Two – Problem of Overfitting

- Overfitting refers to fit the data too well more than is warranted thus may misguide the learning
- Overfitting is not just 'bad generalization' e.g. the VC dimension covers noiseless & noise targets
- Theory of Regularization are approaches against overfitting and prevent it using different methods
 - Key problem: noise in the target function leads to overfitting
 - Effect: 'noisy target function' and its noise misguides the fit in learning
 - There is always 'some noise' in the data
 - Consequence: poor target function ('distribution') approximation
 - Example: Target functions is second order polynomial (i.e. parabola)
 - Using a higher-order polynomial fit
 - lacksquare Perfect fit: low $E_{in}(g)$, but large $E_{out}(g)$



(but simple polynomial works good enough)

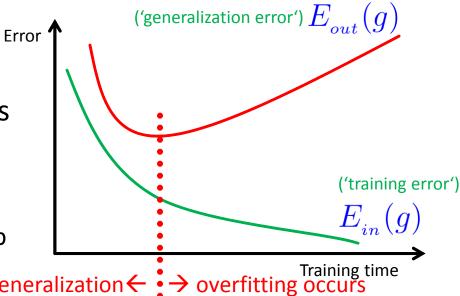
('over': here meant as 4th order,
a 3rd order would be better, 2nd best)

Problem of Overfitting – Clarifying Terms

- A good model must have low training error (E_{in}) and low generalization error (E_{out})
- Model overfitting is if a model fits the data too well (Ein) with a poorer generalization error (Eout) than another model with a higher training error (E_{in})

[1] Introduction to Data Mining

- Overfitting & Errors
 - $\blacksquare E_{in}(g)$ goes down
 - $\blacksquare E_{out}(g)$ goes up
- 'Bad generalization area' ends
 - Good to reduce $E_{in}(g)$
- 'Overfitting area' starts
 - Reducing $E_{in}(g)$ does not help
 - Reason 'fitting the noise' bad generalization ← → overfitting occurs

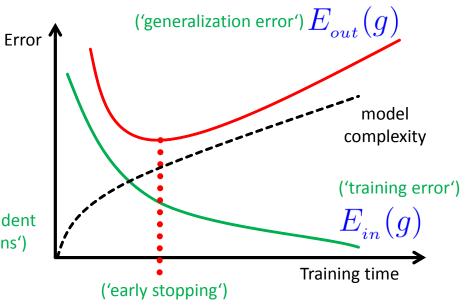


- The two general approaches to prevent overfitting are (1) regularization and (2) validation
- Lecture 6 provides details on validation to be considered as another method against overfitting

Problem of Overfitting – Model Relationships

- Review 'overfitting situations'
 - When comparing 'various models' and related to 'model complexity'
 - Different models are used, e.g. 2nd and 4th order polynomial
 - Same model is used with e.g. two different instances
 (e.g. two neural networks but with different parameters)
- Intuitive solution
 - Detect when it happens
 - 'Early stopping regularization term' to stop the training
 - Early stopping method (later)

('model complexity measure: the VC analysis was independent of a specific target function – bound for all target functions')

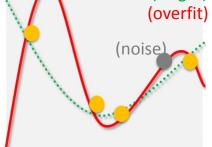


'Early stopping' approach is part of the theory of regularization, but based on validation methods

Problem of Overfitting – Noise Term Revisited

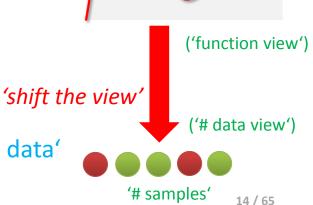
- '(Noisy) Target function' is not a (deterministic) function
 - Getting with 'same x in' the 'same y out' is not always given in practice
 - Idea: Use a 'target distribution' instead of 'target function'
 - Fitting some noise in the data is the basic reason for overfitting and harms the learning process
 - Big datasets tend to have more noise in the data so the overfitting problem might occur even more intense

Unknown Target Distribution $P(y|\mathbf{x})$ target function $f:X \to Y$ plus noise (ideal function)



(target)

- 'Different types of <u>some</u> noise' in data
 - Key to understand overfitting & preventing it
 - 'Shift of view': refinement of noise term
 - Learning from data: 'matching properties of # data'

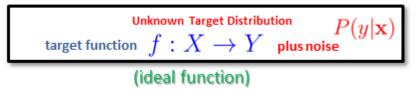


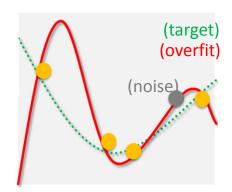
Problem of Overfitting – Stochastic Noise

- Stoachastic noise is a part 'on top of' each learnable function
 - Noise in the data that can not be captured and thus not modelled by f
 - Random noise : aka 'non-deterministic noise'
 - Conventional understanding established early in this course
 - Finding a 'non-existing pattern in noise not feasible in learning'



- Random fluctuations and/or measurement errors in data (cf. Lecture 1, PANGAEA)
- Fitting a pattern that not exists 'out-of-sample'
- Puts learning progress 'off-track' and 'away from f'



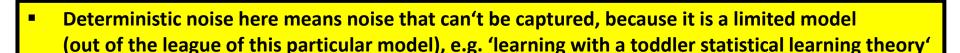


Stochastic noise here means noise that can't be captured, because it's just pure 'noise as is'
(nothing to look for) – aka no pattern in the data to understand or to learn from

Problem of Overfitting – Deterministic Noise

- Part of target function f that H can not capture: $f(\mathbf{x}) h^*(\mathbf{x})$
 - Hypothesis set H is limited so best h* can not fully approximate f
 - h* approximates f, but fails to pick certain parts of the target f
 - Behaves like noise', existing even if data is 'stochastic noiseless'
- Different 'type of noise' than stochastic noise
 - Deterministic noise depends on ${\cal H}$ (determines how much more can be captured by h*)
 - E.g. same f, and more sophisticated \mathcal{H} : noise is smaller (stochastic noise remains the same, nothing can capture it)
 - Fixed for a given x, clearly measurable (stochastic noise may vary for values of x)

(learning deterministic noise is outside the ability to learn for a given h*)



(f)

(h*)

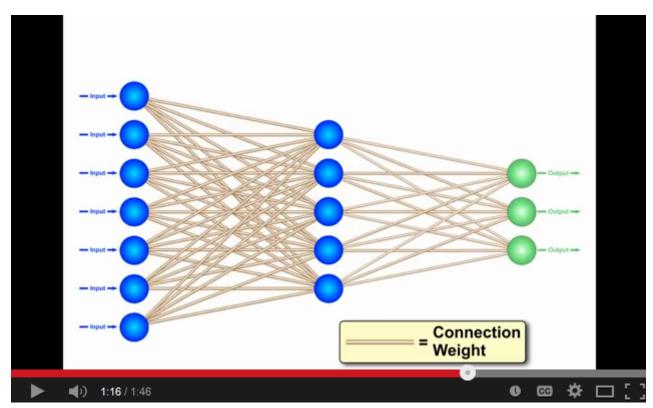
Problem of Overfitting – Impacts on Learning

- The higher the degree of the polynomial (cf. model complexity), the more degrees of freedom are existing and thus the more capacity exists to overfit the training data
- Understanding deterministic noise & target complexity
 - Increasing target complexity increases deterministic noise (at some level)
 - Increasing the number of data N decreases the deterministic noise
- Finite N case: \mathcal{H} tries to fit the noise
 - Fitting the noise straightforward (e.g. Perceptron Learning Algorithm)
 - Stochastic (in data) and deterministic (simple model) noise will be part of it
- Two 'solution methods' for avoiding overfitting
 - Regularization: 'Putting the brakes in learning', e.g. early stopping (more theoretical, hence 'theory of regularization')
 - Validation: 'Checking the bottom line', e.g. other hints for out-of-sample (more practical, methods on data that provides 'hints')
- Lecture 6 provides details on validation to be considered as another method against overfitting

Exercises

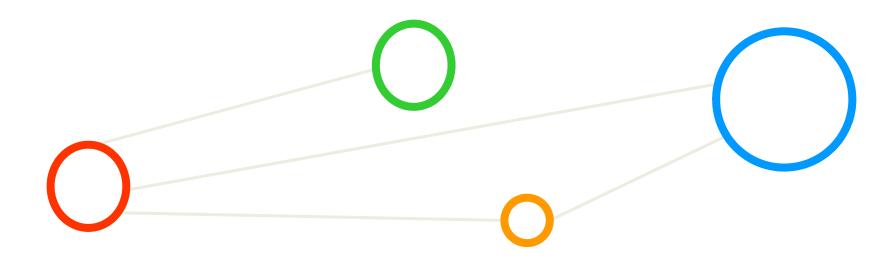


[Video] Towards Multi-Layer Perceptrons



[3] YouTube Video, Neural Networks – A Simple Explanation

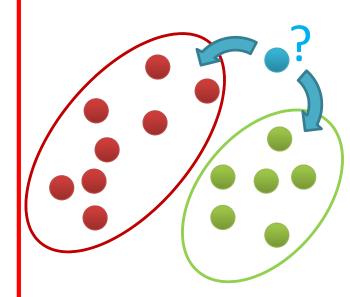
Maximum Margin Classifier



Methods Overview – Focus in this Lecture

Statistical data mining methods can be roughly categorized in classification, clustering, or regression augmented with various techniques for data exploration, selection, or reduction

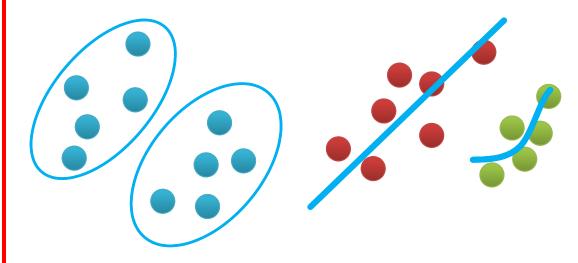
Classification



- Groups of data exist
- New data classified to existing groups

Clustering





- No groups of data exist Create groups from data close to each other
- Identify a line with a certain slope describing the data

Term Support Vector Machines Refined

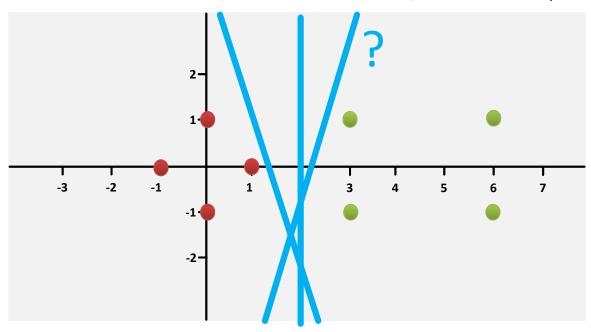
- Support Vector Machines (SVMs) are a classification technique developed ~1990
- SVMs perform well in many settings & are considered as one of the best 'out of the box classifiers'

[2] An Introduction to Statistical Learning

- Term detailed refinement into 'three separate techniques'
 - Practice: applications mostly use the SVMs with kernel methods
- 'Maximal margin classifier'
 - A simple and intuitive classifier with a 'best' linear class boundary
 - Requires that data is 'linearly separable'
- 'Support Vector Classifier'
 - Extension to the maximal margin classifier for non-linearly seperable data
 - Applied to a broader range of cases, idea of 'allowing some error'
- Support Vector Machines → Using Non-Linear Kernel Methods
 - Extension of the support vector classifier
 - Enables non-linear class boundaries & via kernels;

Expected Out-of-Sample Performance for 'Any Line'

- We believe there is a (linear) pattern to be detected
 - Assumption: linearly seperable data (later non-seperable cases)
 - Performance question: What is the optimal line (decision boundary)?
 - **E.g.** green data: $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\}$ red data: $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$



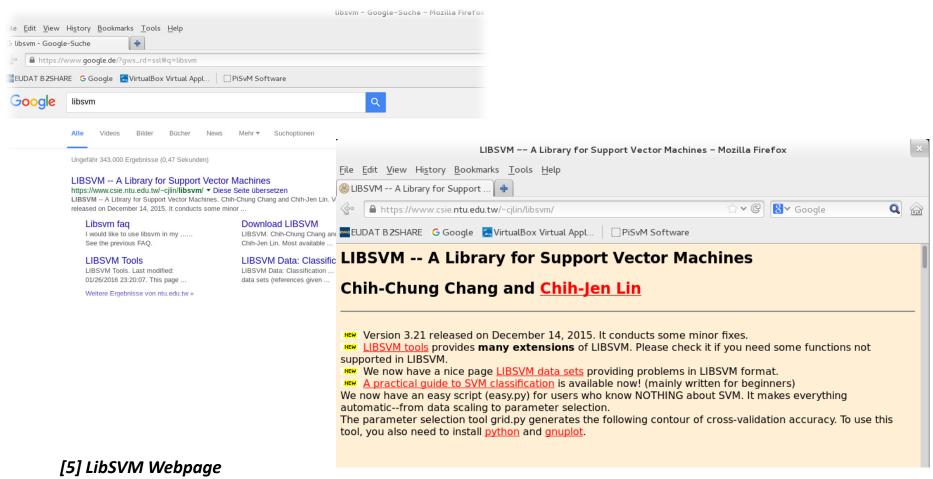
(PLA gives us just any line as soon as all samples are correctly classified)

(How can we craft a margin expressing 'furthest away')

• Intuition tells us just 'furthest away' from the closest points is a good position for the line – why?

LibSVM – Defacto Standard SVM Implementation

- Free available tool
 - Includes Sequential Minimal Optimization (SMO) implementation



LibSVM Installation – Download

Download tar.gz (or in Windows zip bundle)



[5] LibSVM Webpage

- Put package in a folder of your choice
 - Alternatively copy file to your usual working environment

adminuser@linux-8djg:~/tools> scp libsvm-3.21.tar.gz mriedel@jureca.fz-juelich.de:/homeb/zam/mriedel libsvm-3.21.tar.gz 100% 827KB 827.4KB/s 00:00

```
-bash-4.2$ ls -al total 64 drwxr-xr-x 2 mriedel zam 512 Jul 6 20:00 . drwxr-xr-x 29 mriedel zam 32768 Jul 6 19:58 .. -rw-r--r-- 1 mriedel zam 847291 Jul 6 20:00 libsvm-3.21.tar.gz -bash-4.2$ pwd /homeb/zam/mriedel/serialtools
```

LibSVM Installation – Unpack the Bundle

Untar (or Unzip in Windows)

```
/homeb/zam/mriedel/serialtools
-bash-4.2$ tar -zxvf libsvm-3.21.tar.gz
libsvm-3.21/
libsvm-3.21/COPYRIGHT
libsvm-3.21/svm-predict.c
libsvm-3.21/svm.cpp
                                                        /homeb/zam/mriedel/serialtools/libsvm-3.21
libsvm-3.21/README
                                                        -bash-4.2$ ls -al
libsvm-3.21/Makefile.win
                                                        total 544
libsvm-3.21/svm.h
                                                        drwxr-xr-x 8 mriedel zam 32768 Dec 14 2015 .
libsvm-3.21/heart scale
                                                        drwxr-xr-x 3 mriedel zam 512 Jul 6 20:03 ...
libsvm-3.21/iava/
                                                        -rw-r--r-- 1 mriedel zam 1497 Dec 14 2015 COPYRIGHT
libsvm-3.21/java/svm toy.java
                                                        -rw-r--r-- 1 mriedel zam 83089 Dec 14 2015 FAQ.html
libsvm-3.21/java/svm scale.java
                                                        -rw-r--r-- 1 mriedel zam 27670 Dec 14 2015 heart scale
libsvm-3.21/iava/libsvm/
                                                        drwxr-xr-x 3 mriedel zam 512 Dec 14 2015 java
libsvm-3.21/java/libsvm/svm model.java
                                                        -rw-r--r-- 1 mriedel zam 732 Dec 14 2015 Makefile
libsvm-3.21/java/libsvm/svm.m4
                                                        -rw-r--r-- 1 mriedel zam 1136 Dec 14 2015 Makefile.win
libsvm-3.21/java/libsvm/svm problem.java
                                                        drwxr-xr-x 2 mriedel zam 512 Dec 14 2015 matlab
libsvm-3.21/java/libsvm/svm.java
                                                        drwxr-xr-x 2 mriedel zam 512 Dec 14 2015 python
libsvm-3.21/java/libsvm/svm node.java
                                                        -rw-r--r-- 1 mriedel zam 28679 Dec 14 2015 README
libsvm-3.21/java/libsvm/svm parameter.java
                                                         -rw-r--r-- 1 mriedel zam 64836 Dec 14 2015 svm.cpp
libsvm-3.21/java/libsvm/svm print interface.java
                                                         -rw-r--r-- 1 mriedel zam 477 Dec 14 2015 svm.def
libsvm-3.21/java/svm train.java
                                                         -rw-r--r-- 1 mriedel zam 3382 Dec 14 2015 svm.h
libsvm-3.21/java/Makefile
                                                         -rw-r--r-- 1 mriedel zam 5536 Dec 14 2015 sym-predict.c
libsvm-3.21/java/test applet.html
                                                        -rw-r--r-- 1 mriedel zam 8539 Dec 14 2015 svm-scale.c
libsvm-3.21/java/libsvm.jar
                                                        drwxr-xr-x 5 mriedel zam 512 Dec 14 2015 sym-toy
libsvm-3.21/java/svm predict.java
                                                         -rw-r--r-- 1 mriedel zam 8986 Dec 14 2015 svm-train.c
libsvm-3.21/Makefile
                                                        drwxr-xr-x 2 mriedel zam 512 Dec 14 2015 tools
libsvm-3.21/windows/
                                                        drwxr-xr-x 2 mriedel zam 512 Dec 14 2015 windows
libsvm-3.21/windows/svm-tov.exe
libsvm-3.21/windows/svm-scale.exe
libsvm-3.21/windows/svmtrain.mexw64
libsym-3.21/windows/libsymwrite.mexw64
libsvm-3.21/windows/libsvm.dll
```

LibSVM Installation – Make (only in UNIX)

Use make to generate executables (needs g++ compiler)

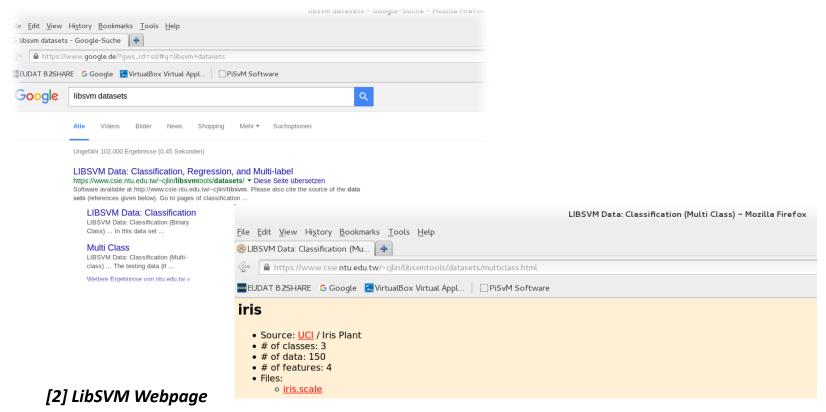
```
-bash-4.2$ pwd
/homeb/zam/mriedel/serialtools/libsvm-3.21
-bash-4.2$ make
g++ -Wall -Wconversion -03 -fPIC -c svm.cpp
g++ -Wall -Wconversion -03 -fPIC svm-train.c svm.o -o svm-train -lm
g++ -Wall -Wconversion -03 -fPIC svm-predict.c svm.o -o svm-predict -lm
q++ -Wall -Wconversion -03 -fPIC svm-scale.c -o svm-scale
```

Check executables important for us

```
-bash-4.2$ pwd
/homeb/zam/mriedel/serialtools/libsvm-3.21
-bash-4.2$ ls -al
drwxr-xr-x 8 mriedel zam 32768 Jul 6 20:05 .
drwxr-xr-x 3 mriedel zam 512 Jul 6 20:03 ...
-rw-r--r-- 1 mriedel zam 1497 Dec 14 2015 COPYRIGHT
-rw-r--r-- 1 mriedel zam 83089 Dec 14 2015 FAQ.html
-rw-r--r-- 1 mriedel zam 27670 Dec 14 2015 heart scale
drwxr-xr-x 3 mriedel zam 512 Dec 14 2015 java
                        732 Dec 14 2015 Makefile
-rw-r--r-- 1 mriedel zam
-rw-r--r-- 1 mriedel zam 1136 Dec 14 2015 Makefile.win
                         512 Dec 14 2015 matlab
drwxr-xr-x 2 mriedel zam
drwxr-xr-x 2 mriedel zam
                        512 Dec 14 2015 python
-rw-r--r-- 1 mriedel zam 28679 Dec 14 2015 README
-rw-r--r-- 1 mriedel zam 64836 Dec 14 2015 svm.cpp
-rw-r--r-- 1 mriedel zam
                         477 Dec 14 2015 svm.def
-rw-r--r-- 1 mriedel zam 3382 Dec 14 2015 svm.h
                                                          (use in testing phase)
-rwxr-xr-x 1 mriedel zam
                         78270 Jul 6 20:05 svm-predict
-rwxr-xr-x 1 mriedel zam 18587 Jul 6 20:05 svm-scale
-rw-r--r-- 1 mriedel zam
                         8539 Dec 14 2015 svm-scale.c
                                                          (use in training phase)
                        78509 Jul 6 20:05 svm-train
-rwxr-xr-x 1 mriedel zam
drwxr-xr-x 2 mriedel zam
                         512 Dec 14 2015 tools
drwxr-xr-x 2 mriedel zam
                         512 Dec 14 2015 windows
```

Scaling

- Scaled version of our data (cf. Lecture 1 & 3): iris.scale
 - Scaling is used in order that the optimization does not have to work with large numbers – so one can scale, but it is not a requirement
 - Sometimes the performance improved with scaling



Data Preparation Phase

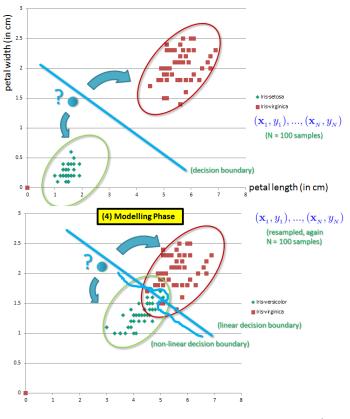
Copy IRIS Dataset in your working environment

adminuser@linux-8djg:~/data> scp iris.scale mriedel@jureca.fz-juelich.de:/homeb/zam/mriedeliris.scale 100% 6954

6.8KB/s 00:00

```
/homeb/zam/mriedel/datasets
-bash-4.2$ ls -al
total 64
drwxr-xr-x 2 mriedel zam 512 Jul 6 21:53 .
drwxr-xr-x 30 mriedel zam 32768 Jul 6 21:53 ..
-rw-r--r- 1 mriedel zam 6954 Jul 6 21:53 iris.scale
```

- Dataset Two-class problem, linearly seperable
 - Dataset Iris Setosa (class 1) and Iris Virginica (class 3)
 - Iris-class1and3-training(20)/testing(30)
- Dataset Two-class problem, not linearly seperable
 - Dataset Iris Veriscolor (class 2) and Iris Virginica (class3)
 - iris-class2and3-training(20)/testing(30)



Exercises



Iris Dataset

Iris dataset is already available in the tuturial directory

Iris Dataset LibSVM Format Preprocessing

Morris Riedel ; 03 July 2016 http://bzshare.eudat.eu

Abstract: UCI Machine Learning Repository IRIS Dataset

iris.scale.original and iris.scale

- 3 classes, 50 samples each class iris-class1and3
- only linearly seperable data
- class 1 and 3 sampling
- 100 samples

iris-class1and3-training/testing

- 20 for training, 30 for testing
- per class 1 and 3

iris-class2and3-training/testing

- 20 for training, 30 for testing
- per class 2 and 3

(persistent handle link for publication into papers)

**B2SHARE Store and Share Research Date of the Publication into papers | Compare Notice | Compare Notice

[6] Iris Dataset LibSVM Format Preprocessing

Keyword(s): LibSVM; Iris; Flowers; UCI

The record appears in these collections:

Generic

```
/apps/gent/tutorials/machine learning/classification/Iris
[vsc42544@gligar02 Iris]$ ls -al
total 256
drwxr-xr-x 2 vsc40003 vsc40003 4096 Nov 22 15:42 .
drwxr-xr-x 6 vsc40003 vsc40003 4096 Nov 22 15:44 ...
-rw-r--r-- 1 vsc40003 vsc40003 2736 Nov 9 21:41 iris-class1and3-testing.txt
-rw-r--r-- 1 vsc40003 vsc40003 1806 Nov
                                        9 21:41 iris-class1and3-training.txt
-rw-r--r-- 1 vsc40003 vsc40003 4542 Nov
                                        9 21:41 iris-classland3.txt
-rw-r--r-- 1 vsc40003 vsc40003 2841 Nov 9 21:41 iris-class2and3-testing.txt
-rw-r--r-- 1 vsc40003 vsc40003 3184 Nov 9 21:41 iris-class2and3-training.txt
                                        9 21:42 iris-class2and3.txt
-rw-r--r-- 1 vsc40003 vsc40003 4658 Nov
-rw-r--r-- 1 vsc40003 vsc40003 6954 Nov 9 21:42 iris.scale.original.original
-rw-r--r-- 1 vsc40003 vsc40003 6954 Nov 9 21:42 iris.scale.scale
```

Training Phase: linearly seperable case (iris-class1and3)

Use svm-train (c<=0 not allowed)

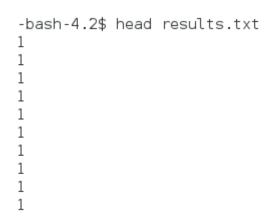
```
-bash-4.2$ more svm-train1-3.sh
     ./svm-train -t 0 -c 1 /homeb/zam/mriedel/datasets/iris-classland3-training
     -bash-4.2$ ./svm-train1-3.sh
                                                     -bash-4.2$ ls -al
    optimization finished, #iter = 11
                                                     total 896
    nu = 0.035490
                                                                               32768 Jul 6 22:25 .
                                                     drwxr-xr-x 8 mriedel zam
    obi = -0.709742, rho = 0.447384
                                                     drwxr-xr-x 3 mriedel zam
                                                                                 512 Jul 6 20:03 ...
                                                                                1497 Dec 14 2015 COPYRIGHT
                                                     -rw-r--r-- 1 mriedel zam
    nSV = 4, nBSV = 0
                                                                               83089 Dec 14 2015 FAO.html
                                                      -rw-r--r-- 1 mriedel zam
    Total nSV = 4
                                                      -rw-r--r-- 1 mriedel zam
                                                                                  354 Jul 6 22:25 iris-class1and3-training.model
                                                                                  312 Dec 14 2013 java
                                                     -rw-r--r-- 1 mriedel zam
                                                                                  732 Dec 14 2015 Makefile
         Check model file
                                                     -rw-r--r-- 1 mriedel zam
                                                                                1136 Dec 14 2015 Makefile.win
                                                     drwxr-xr-x 2 mriedel zam
                                                                                 512 Dec 14 2015 matlab
                                                     drwxr-xr-x 2 mriedel zam
                                                                                 512 Dec 14
                                                                                             2015 python
                                                     -rw-r--r-- 1 mriedel zam
                                                                               28679 Dec 14
                                                                                             2015 README
                                                     -rw-r--r-- 1 mriedel zam
                                                                               64836 Dec 14
                                                                                             2015 svm.cpp
                                                                                             2015 svm.def
                                                     -rw-r--r-- 1 mriedel zam
                                                                                 477 Dec 14
-bash-4.2$ more iris-class1and3-training.model
                                                                                3382 Dec 14 2015 svm.h
                                                     -rw-r--r-- 1 mriedel zam
svm type c svc
                                                     -rw-r--r-- 1 mriedel zam 100224 Jul 6 20:05 svm.o
kernel type linear
                                                     -rwxr-xr-x 1 mriedel zam
                                                                               78270 Jul 6 20:05 svm-predict
nr class 2
total sv 4
                                                     -rw-r--r-- 1 mriedel zam
                                                                                5536 Dec 14 2015 svm-predict.c
rho 0.447384
                                                     -rwxr-xr-x 1 mriedel zam
                                                                               18587 Jul 6 20:05 sym-scale
label 1 3
                                                     -rw-r--r-- 1 mriedel zam
                                                                                8539 Dec 14 2015 svm-scale.c
nr sv 3 1
                                                     drwxr-xr-x 5 mriedel zam
                                                                                 512 Dec 14 2015 sym-toy
0.1374686716356165 1:-0.666667 2:-0.166667 3:-0.864407 4:-0.916667 -rwxr-xr-x 1 mriedel zam
                                                                               78509 Jul 6 20:05 svm-train
0.1011680329343598 1:-0.388889 2:0.583333 3:-0.762712 4:-0.75
                                                      -rwxr-xr-x 1 mriedel zam
                                                                                  76 Jul 6 22:24 svm-train1-3.sh
0.4711540941141194 1:-0.944444 2:-0.25 3:-0.864407 4:-0.916667
-0.7097907986840956 1:-0.666667 2:-0.583333 3:0.186441 4:0.333333 -rw-r--r-- 1 mriedel zam
                                                                                8986 Dec 14 2015 svm-train.c
                                                     drwxr-xr-x 2 mriedel zam
                                                                                 512 Dec 14 2015 tools
                                                     drwxr-xr-x 2 mriedel zam
                                                                                 512 Dec 14 2015 windows
```

Testing Phase: linearly seperable case (iris-class1and3)

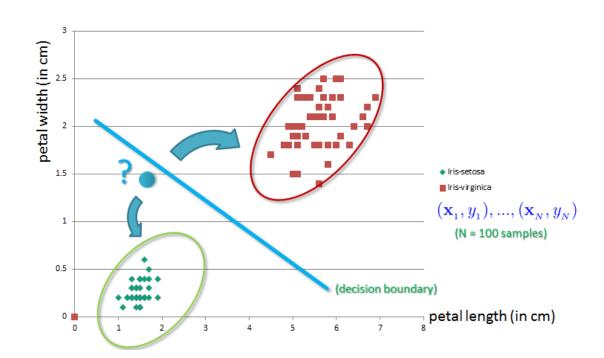
Use svm-predict (using newly created model file & testing data)

```
-bash-4.2$ more svm-predict1-3.sh
./svm-predict /homeb/zam/mriedel/datasets/iris-class1and3-testing ./iris-class1and3-training.model ./results.txt
```

```
-bash-4.2$ ./svm-predict1-3.sh
Accuracy = 100% (60/60) (classification)
```



(consistent with our graph: 100% here possible since very easy problem, in practice rarely)



LibSVM – svm-train Parameters

Important parameters (training phase)

```
-bash-4.2$ ./svm-train
Usage: svm-train [options] training set file [model file]
                                                                 (we need a training set file)
                                                                 (take default here = C-SVC)
-s svm type : set type of SVM (default 0)
        0 -- C-SVC
                                 (multi-class classification
        1 -- nu-SVC
                                  (multi-class classification)
        2 -- one-class SVM
        3 -- epsilon-SVR
                                  (regression)
-t kernel type : set type of kernel function (default 2)
                                                                 (in this lecture we have just 'linear kernels')
        0 -- linear: u'*v
        1 -- polynomial: (gamma*u'*v + coef0)^degree
        2 -- radial basis function: exp(-gamma*|u-v|^2)
        3 -- sigmoid: tanh(gamma*u'*v + coef0)
        4 -- precomputed kernel (kernel values in training set file)
-d degree : set degree in kernel function (default 3)
-g gamma : set gamma in kernel function (default 1/num features)
-r coef0 : set coef0 in kernel function (default 0)
-c cost : set the parameter C of C-SVC, epsilon-SVR, and nu-SVR (default 1)
                                                                                 (Regularization Parameter)
-n nu : set the parameter nu of nu-SVC, one-class SVM, and nu-SVR (default 0.5)
-p epsilon: set the epsilon in loss function of epsilon-SVR (default 0.1)
-m cachesize : set cache memory size in MB (default 100)
-e epsilon : set tolerance of termination criterion (default 0.001)
-h shrinking : whether to use the shrinking heuristics, 0 or 1 (default 1)
-b probability estimates : whether to train a SVC or SVR model for probability estimates, 0 or 1 (default 0)
-wi weight : set the parameter C of class i to weight*C, for C-SVC (default 1)
-v n: n-fold cross validation mode
-q : quiet mode (no outputs)
                                                                 Training Examples
                                                            (\mathbf{x}_{_{1}},y_{_{1}}),...,(\mathbf{x}_{_{N}},y_{_{N}})
```

LibSVM – svm-predict Parameters

Important parameters (testing phase)

-hash-4.2\$./svm-predict
Usage: svm-predict [options] test_file model_file output_file

-b probability_estimates: whether to predict probability estimates, 0 or 1 (default 0); for one-class SVM only 0 is supported -q : quiet mode (no outputs)

(the model file is generated in the training phase \rightarrow the support vectors found in optimization)

(test file is a testing dataset set aside to be used once training is finished)

(output file gives us indications how each sample was classified)

Testing Examples $(\mathbf{x}_{\!\scriptscriptstyle 1},y_{\!\scriptscriptstyle 1}),...,(\mathbf{x}_{\!\scriptscriptstyle N},y_{\!\scriptscriptstyle N})$

Interactive Session on Golett for Serial LibSVM

- Use "qsub -I -W x=FLAGS:ADVRES:<reservationID>"
 - (the -I option here is 'minus capital i')
 - Running 'module swap cluster/golett' first
 - qsub -I -W x=FLAGS:ADVRES:machine_learning.317 (Friday)
 - Note that job scripts and interactive sessions will by default be allocated
 1 hour of walltime and 1 single processor core
 - Module load LIBSVM/3.22-intel-2016b is our serial SVM implementation

```
[vsc42544@gligar02 ~]$ module swap cluster/golett
The following have been reloaded with a version change:
    1) cluster/delcatty => cluster/golett

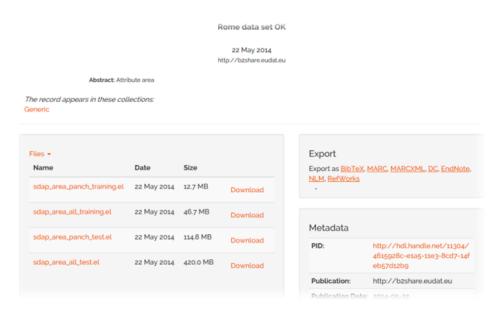
[vsc42544@gligar02 ~]$ qsub -I -W x=FLAGS:ADVRES:machine_learning.317
qsub: waiting for job 1174756.master19.golett.gent.vsc to start

[vsc42544@gligar02 ~]$ qsub -I
qsub: waiting for job 1174757.master19.golett.gent.vsc to start
qsub: job 1174757.master19.golett.gent.vsc ready

[vsc42544@node2400 ~]$ module load LIBSVM/3.22-intel-2016b
```

Rome Remote Sensing Dataset (cf. Lecture 3)

Data is already available in the tutorial directory



(persistent handle link for publication into papers)



[8] Rome Image dataset

Indian Pines Remote Sensing Dataset (cf. Lecture 3)

Indian Pines Dataset Raw and Processed

(persistent handle link for publication into papers)

Abstract: 1) Indian raw: 1417x614x200 (training 10% and test)
2) Indian processed:1417x614x30 (training 10% and test)

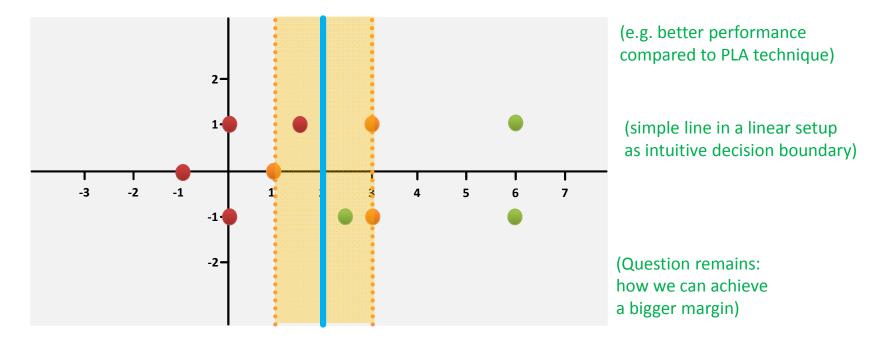
Name	Date	Size	
indian_processed_training.el	05 Feb 2015	11.7 MB	Download
indian_raw_test.el	05 Feb 2015	747.1 MB	Download
indian_raw_training.el	05 Feb 2015	83.0 MB	Download
indian_processed_test.el	05 Feb 2015	105.6 MB	Download



[9] Indian Pine Image dataset

Expected Out-of-Sample Performance for 'Best Line'

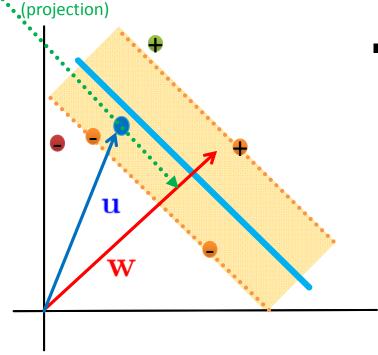
- The line with a 'bigger margin' seems to be better but why?
 - Intuition: chance is higher that a new point will still be correctly classified
 - Fewer hypothesis possible: constrained by sized margin (cf. Lecture 3)
 - Idea: achieving good 'out-of-sample' performance is goal (cf. Lecture 3)



Support Vector Machines (SVMs) use maximum margins that will be mathematically established

Geometric SVM Interpretation and Setup (1)

- Think 'simplified coordinate system' and use 'Linear Algebra'
 - Many other samples are removed (red and green not SVs)
 - Vector w of 'any length' perpendicular to the decision boundary
 - Vector u points to an unknown quantity (e.g. new sample to classify)
 - Is u on the left or right side of the decision boundary?



■ Dot product $\mathbf{w} \cdot \mathbf{u} \ge C$; C = -b

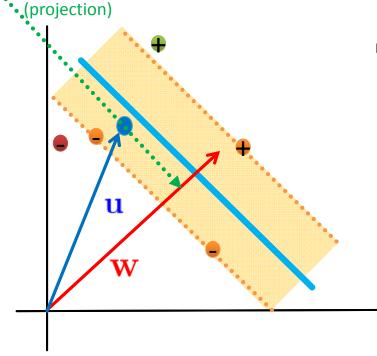
- With u takes the projection on the W
- Depending on where projection is it is left or right from the decision boundary
- Simple transformation brings decison rule:
- $\mathbf{u} \cdot \mathbf{u} + b \ge 0 \rightarrow \mathsf{means} +$
- (given that b and W are unknown to us)

(constraints are not enough to fix particular b or w, need more constraints to calculate b or w)

Geometric SVM Interpretation and Setup (2)

- Creating our constraints to get b or w computed
 - First constraint set for positive samples \bullet $\mathbf{w} \cdot \mathbf{x}_+ + b \geq 1$
 - Second constraint set for negative samples $\mathbf{w} \cdot \mathbf{x}_- + b \leq 1$
 - For mathematical convenience introduce variables (i.e. labelled samples)

$$y_i = +$$
 for \bullet and $y_i = -$ for \bullet



Multiply equations by y_i

• Positive samples: $y_i(\mathbf{x}_i \cdot \mathbf{w} + b) \ge 1$

• Negative samples: $y_i(\mathbf{x}_i \cdot \mathbf{w} + b) \ge 1$

■ Both same due to $y_i = +$ and $y_i = -$ (brings us mathematical convenience often quoted)

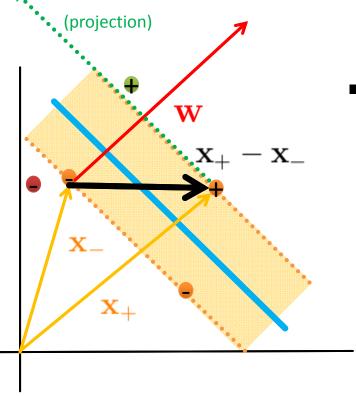
$$y_i(\mathbf{x}_i \cdot \mathbf{w} + b) - 1 \ge 0$$

(additional constraints just for support vectors itself helps)

$$2 y_i(\mathbf{x}_i \cdot \mathbf{w} + b) - 1 = 0$$

Geometric SVM Interpretation and Setup (3)

- Determine the 'width of the margin'
 - Difference between positive and negative SVs: $\mathbf{x}_+ \mathbf{x}_-$
 - Projection of $\mathbf{x}_+ \mathbf{x}_-$ onto the vector \mathbf{w}
 - The vector \mathbf{W} is a normal vector, magnitude is $\|\mathbf{w}\|$



(Dot product of two vectors is a scalar, here the width of the margin)

- Unit vector is helpful for 'margin width'
 - Projection (dot product) for margin width:

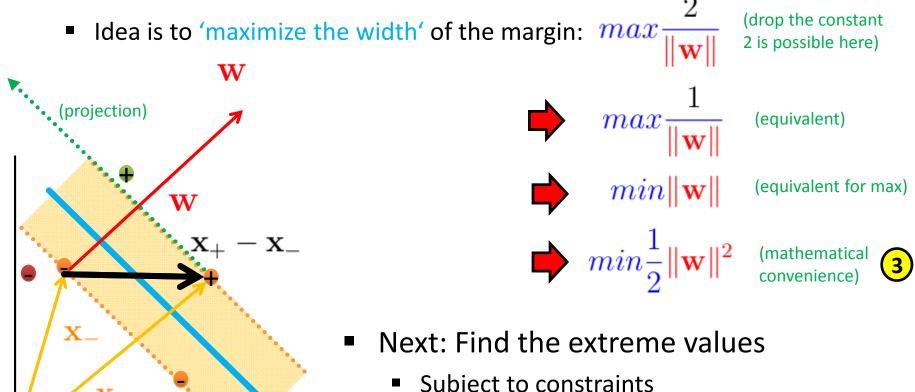
$$\mathbf{x}_{+} - \mathbf{x}_{-} \cdot \frac{\mathbf{w}}{\|\mathbf{w}\|}$$
 (unit vector)
$$1 - b \quad 1 + b$$

- When enforce constraint:
- (2) $y_i(\mathbf{x}_i \cdot \mathbf{w} + b) 1 = 0$ $y_i = -6$

$$y_i = -$$

Constrained Optimization Steps SVM (1)

- Use 'constraint optimization' of mathematical toolkit

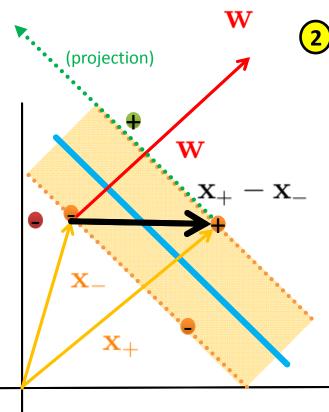


- Subject to constraints
 - $y_i(\mathbf{x}_i \cdot \mathbf{w} + b) 1 = 0$

Constrained Optimization Steps SVM (2)

- Use 'Lagrange Multipliers' of mathematical toolkit
 - Established tool in 'constrained optimization' to find function extremum
 - 'Get rid' of constraints by using Lagrange Multipliers





$$\mathbf{2} y_i(\mathbf{x}_i \cdot \mathbf{w} + b - 1) = 0$$

Introduce a multiplier for each constraint

$$\mathcal{L}(\alpha) = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^{n} \alpha_i [y_i(\mathbf{x}_i \cdot \mathbf{w} + b) - 1]$$

(interesting: non zero for support vectors, rest zero)

- Find derivatives for extremum & set 0
 - But two unknowns that might vary
 - First differentiate w.r.t. W
 - Second differentiate w.r.t. b

(derivative gives the gradient, setting 0 means extremum like min)

Constrained Optimization Steps SVM (3)

■ Lagrange gives:
$$\mathcal{L}(\alpha) = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^{n} \alpha_i [y_i(\mathbf{x}_i \cdot \mathbf{w} + b) - 1]$$

First differentiate w.r.t w

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \mathbf{w} - \sum \alpha_i y_i \mathbf{x}_i = 0$$
 (derivative gives the gradient, setting 0 means extremum like min)

Simple transformation brings:

$$\mathbf{5} \mathbf{w} = \sum_{i=1}^{n} \alpha_i y_i \mathbf{x}_i \quad \text{(i.e. vector is } \underline{\text{linear sum of samples}})$$

(recall: non zero for support vectors, rest zero → even less samples)

Second differentiate w.r.t. b

$$\frac{\partial \mathcal{L}}{\partial b} = -\sum \alpha_i y_i = 0 \implies \sum \alpha_i y_i = 0$$
 5

(projection)

Constrained Optimization Steps SVM (4)

Lagrange gives:
$$\mathcal{L}(\alpha) = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^{n} \alpha_i [y_i(\mathbf{x}_i \cdot \mathbf{w} + b) - 1]$$





• Take advantage of
$$\mathbf{5}\mathbf{w} = \sum \alpha_i y_i \mathbf{x}_i$$

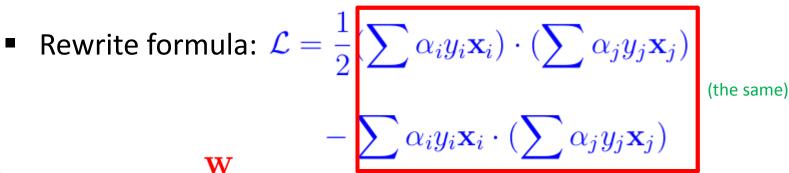
$$\mathcal{L} = \frac{1}{2} \left(\sum \alpha_i y_i \mathbf{x}_i \right) \cdot \left(\sum \alpha_j y_j \mathbf{x}_j \right)$$

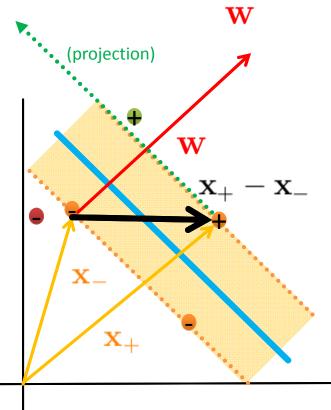
$$-\sum \alpha_i y_i \mathbf{x}_i \cdot (\sum \alpha_j y_j \mathbf{x}_j)$$

$$-\sum \alpha_i y_i b + \sum \alpha_i$$



Constrained Optimization Steps SVM (5)





$$-\sum_{\text{(was 0)}} \alpha_i y_i b + \sum_{\text{(results in)}} \alpha_i$$

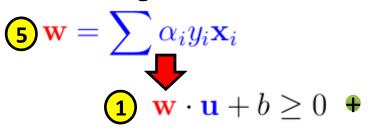
(optimization depends only on dot product of samples)

$$\mathcal{L} = \sum_{i} \alpha_{i} - \frac{1}{2} \sum_{i} \sum_{j} \alpha_{i} \alpha_{j} y_{i} y_{j} \mathbf{x}_{i} \cdot \mathbf{x}_{j}$$

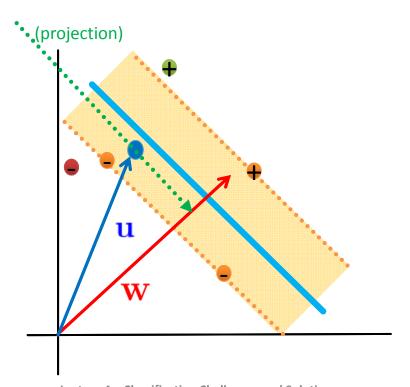
 Equation to be solved by some quadratic programming package

Use of SVM Classifier to Perform Classification

Use findings for decision rule



(decision rule also depends on dotproduct)



Maximal Margin Classifier – Training Set and Test Set

Classification technique

- Given 'labelled dataset'
- Data matrix X (n x p)
- Training set:n training samples
- p-dimensional space
- Linearly seperable data
- Binary classification problem (two class classification)
- Test set: a vector x* with test observations

$$x_1 = \begin{pmatrix} x_{11} \\ \vdots \\ x_{1p} \end{pmatrix}, \dots, x_n = \begin{pmatrix} x_{n1} \\ \vdots \\ x_{np} \end{pmatrix}$$

(n x p-dimensional vectors)

$$y_1, \dots, y_n \in \{-1, 1\}$$
 (class labels) (two classes)

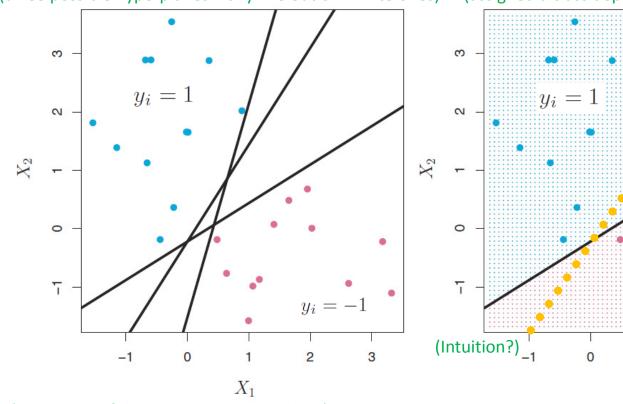
$$x^* = \begin{pmatrix} x_1^* & \dots & x_p^* \end{pmatrix}^T$$

Maximal Margin Classifiers create a seperating hyperplane that seperates the training set samples
perfectly according to their class labels following a reasonable way of which hyperplane to use

Maximal Margin Classifier – Use of Seperating Hyperplanes

(three possible hyperplanes – any line out of infinite ones)

(assigned a class depending on which side of hyperplane)



 $y_i = 1$ (linear decision boundary) $y_i = -1$ (but is this the best line...?) Intuition?) $y_i = -1$ X_1

(properties of the seperating hyperplane)

$$\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} > 0 \text{ if } y_i = 1$$

$$\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} < 0 \text{ if } y_i = -1$$

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip}) > 0 \text{ for all } i = 1, \dots, n$$

modified from [2] An Introduction to Statistical Learning

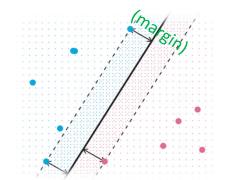
(using testset to predict and assign labels via sign)

sign of
$$f(x^*) = \beta_0 + \beta_1 x_1^* + \beta_2 x_2^* + \ldots + \beta_p x_p^*$$

 $f(x^*)$ is negative assign it to class -1
 $f(x^*)$ is positive assign it to class -1

Maximal Margin Classifier – Reasoning and Margin Term

- Reasoning to pick the 'best line'
 - There exists a 'maximal margin hyperplane' (optimal seperating hyperplane)
 - Hyperplane that is 'farthest away' from the training set samples
- Identify the 'margin' itself
 - Compute the 'perpendicular distance' (point 'right angle 90 degrees' distance to the plane)
 - From each training sample to a given separating hyperplane
 - The smallest such distance is the 'minimal distance' from the observations to the hyperplane the margin
- Identify 'maximal margin'
 - Identify the hyperplane that has the 'farthest minimum distance' to the training observations
 - Also named the 'optimal seperating hyperplane'



The maximal margin hyperplane is the seperating hyperplane for which the margin is largest

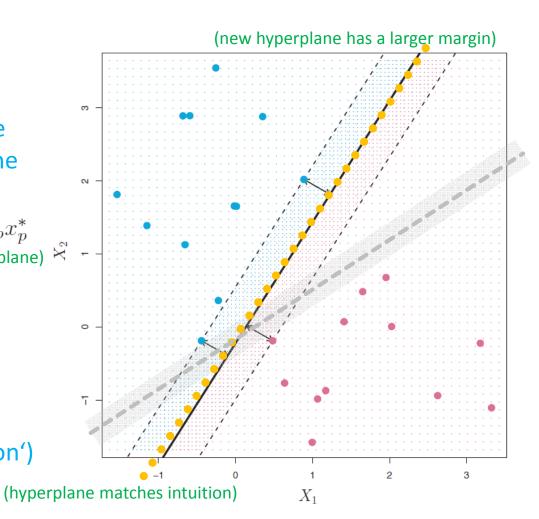
[2] An Introduction to Statistical Learning

Maximal Margin Classifier – Margin Performance

- Classification technique
 - Classify testset samples based on which side of the maximal margin hyperplane they are lying

$$sign of \ f(x^*) = \beta_0 + \beta_1 x_1^* + \beta_2 x_2^* + \ldots + \beta_p x_p^*$$
 (Rs are the coefficients of the maximal margin hyperplane) \aleph

- Assuming that a classifier that has a large margin on the training data will also have a large margin on the test data (cf. also 'the intuitive notion')
- Testset samples will be thus correctly classified



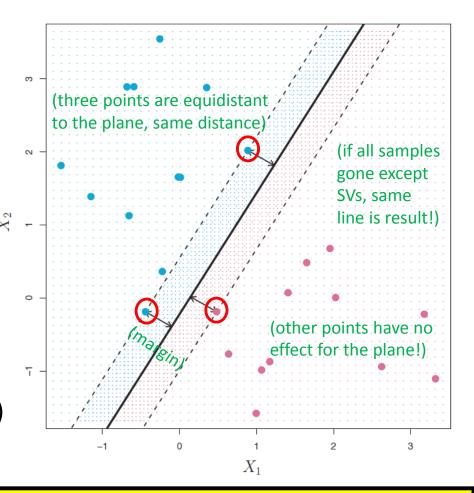
(Compared to grey hyperplane: a 'greater minimal distance' between the data points and the seperating hyperplane)

modified from [2] An Introduction to Statistical Learning

Maximal Margin Classifier – Support Vector Term

Observation

- Three data points
 lie on the edge of margin
 (somewhat special data points)
- Dashed lines indicating the width of the margin (very interesting to know)
- Margin width is the distance from the special data points to the hyperplane (hyperplane depends directly on small data subset: SV points)

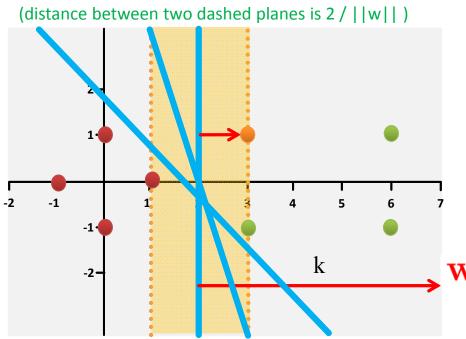


modified from [2] An Introduction to Statistical Learning

- Points that lie on the edge of the margin are named support vectors (SVs) in p-dimensional space
- SVs 'support' the maximal margin hyperplane: if SVs are moved → the hyperplane moves as well

Maximal Margin Classifier - Optimization and W Vector

- Which weight w maximizes the margin?
 - Margin is just a distance from 'a line to a point', goal is to minimize W
 - Pick x_n as the nearest data point to the line (or hyper-plane)...



$$\underset{\beta_0,\beta_1,...,\beta_p}{\text{maximize}} M$$

subject to
$$\sum_{j=1}^{p} \beta_j^2 = 1$$

(for points on plane w must be 0, interpret k as length of w)

$$\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip} = 0$$
 $k(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip}) = 0$ for any $k \neq 0$

Support vectors achieve the margin and are positioned exactly on the boundary of the margin

Maximal Margin Classifier – Optimization and N Samples

- Approach: Maximizing the margin
 - Equivalent to minimize objective function (original and modified have same w and b)

$$\min_{\mathbf{w}_{i}, b} \left\{ \frac{1}{2} ||\mathbf{w}||^{2} \right\} \text{ (substituted for plain } ||\mathbf{w}|| \text{ for mathematical convenience)} \\
\text{subject to } y_{i} \left(\mathbf{w} \cdot \mathbf{x}_{i} - \mathbf{b} \right) >= 1$$

- 'Lagrangian Dual problem' (chain of math turns optimization problem into solving this)
 - Use of already established Lagrangian method :

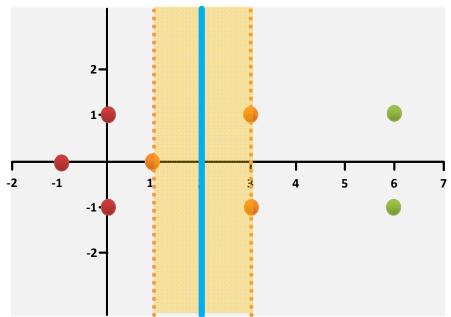
$$\mathcal{L}(\alpha) = \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} y_n y_m \alpha_n \alpha_m \mathbf{x}_n^T \mathbf{x}_m$$
(big data impact, important dot product)

- Interesting properties
 - Simple function: Quadratic in alpha
 - Simple constraints in this optimization problem (not covered here)
 - Established tools exist: Quadratic Programming (qp)
- Practice shows that #N moderate is ok, but large #N ('big data') are problematic for computing
- Quadratic programming and computing the solving depends on number of samples N in the dataset

Maximal Margin Classifier – Optimization and # SV Impacts

- Interpretation of QP results (vector of alpha is returned)
 - The obtained values of alpha (lagrange multipliers) are mostly 0
 - Only a couple of alphas are > 0 and special: the support vectors (SVs)

(three support vectors create optimal line)



- N x N, usually not sparse
- Computational complexity relies in the following:

(big data challenge) (e.g. all datasets vs. sampling)

$$\mathcal{L}(\alpha) = \sum_{n=1}^{N} \alpha_n - \frac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} y_n y_m \alpha_n \alpha_m \mathbf{x}_n^T \mathbf{x}_m$$

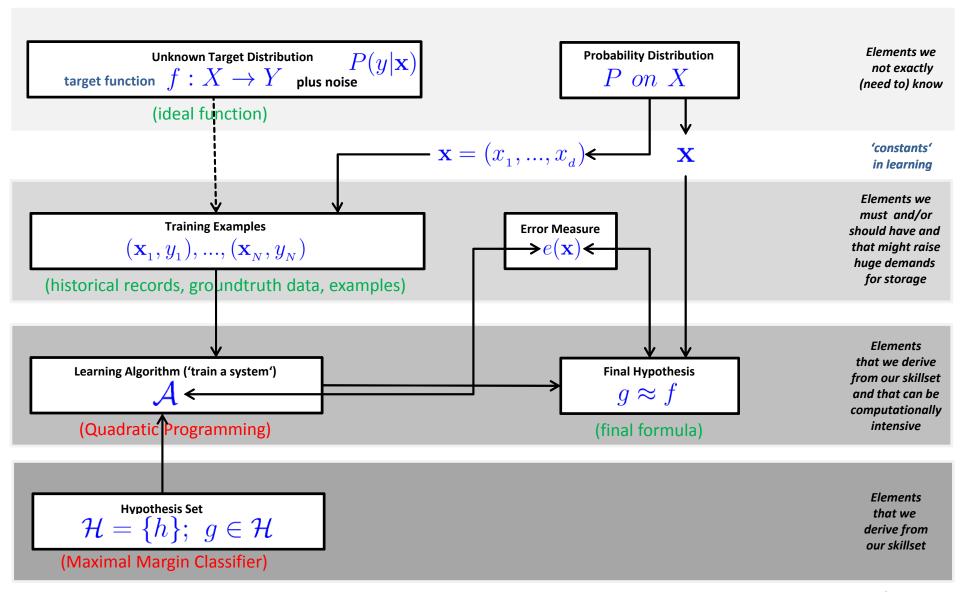
$$\begin{bmatrix} y_1 y_1 x_1^T x_1 & y_1 y_2 x_1^T x_2 & \dots y_1 y_N x_1^T x_N \\ \dots & \dots & \ddots \\ y_N y_1 x_N^T x_1 & y_N y_2 x_N^T x_2 & \dots y_N y_N x_N^T x_N \end{bmatrix}$$

(quadratic coefficients, alphas are result from QP)

(rule of thumb)

- Generalization measure: #SVs as 'in-sample quantity' → 10SVs/1000 samples ok, 500SVs/1000 bad
- Reasonsing towards overfitting due to a large number of SVs (fit many, small margin, gives bad E_{out})

Solution Tools: Maximal Margin Classifier & QP Algorithm



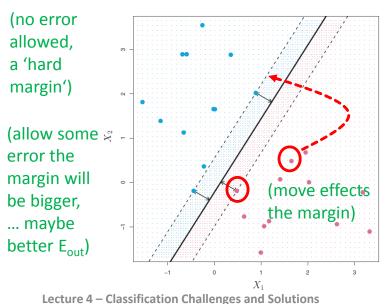
Maximal Margin Classifier – Solving and Limitations

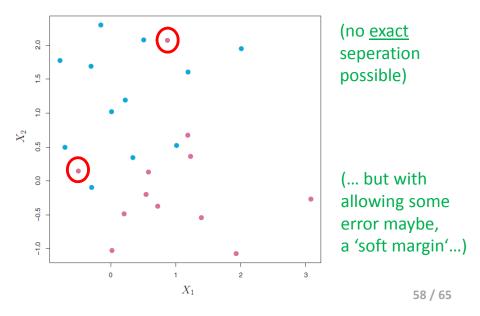
- Solving constraint optimization problem chooses coefficients that maximize M & gives hyperplane
- Solving this problem efficiently is possible techniques like sequential minimal optimization (SMO)
- Maximal margin classifiers use a hard-margin & thus only work with exact linearly seperable data

modified from [2] An Introduction to Statistical Learning

Limitation

- Non linearly separable data (given mostly in practice)
- Optimization problem has no solution M > 0 (think point moves over plane)
- No separating hyperplane can be created (classifier can not be used)





Exercises



Training Indian Pines on Golett – Job Script

- Use Indian Pines and start changing parameters
 - Parameters are equal to the serial libsvm and some additional parameters for paralellization

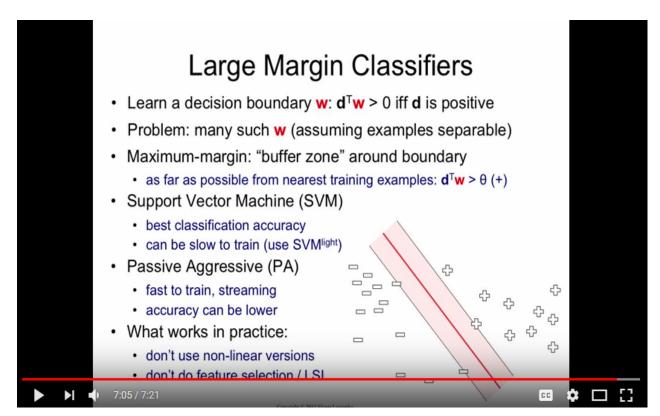
```
#!/bin/bash
#PBS -l walltime=1:0:0
#PBS -l nodes=1:ppn=all
#module load HPDBSCAN/20171110-foss-2017b
module load piSvM-JSC/1.2-20150622-intel-2017b
module load vsc-mympirun
export WORKDIR=$VSC SCRATCH/$PBS JOBID
mkdir -p $WORKDIR
cd $WORKDIR
# Train data
cp /apps/gent/tutorials/machine learning/classification/Indian/indian processed training.el .
# by default, mympirun will use all available cores
# use --hybrid to only use a certain number of cores (per workernode)
# mympirun --hybrid 6 dbscan -e 300 -m 100 -t 12 bremenSmall.h5.h5
mympirun --hybrid 32 pisvm-train -D -o 1025 -q 512 -c 10 -g 8 -t 2 -m 1024 -s 0 indian processed training.el
echo "Results available in $WORKDIR"
```

Testing Indian Pines on Golett – Job Script

- Use Indian Pines and using your model files
 - Parameters are equal to the serial libsvm and some additional parameters for paralellization

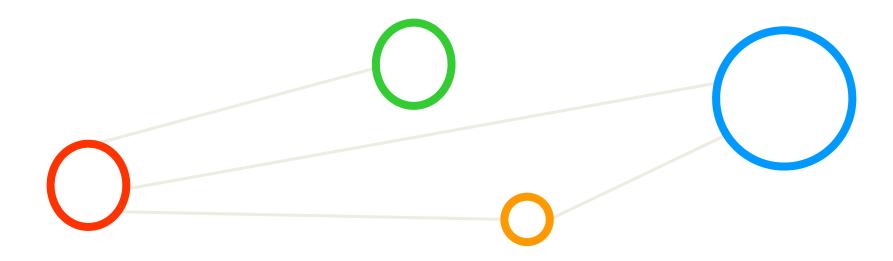
```
#!/bin/bash
#PBS -l walltime=1:0:0
#PBS -l nodes=1:ppn=all
#module load HPDBSCAN/20171110-foss-2017b
module load piSvM-JSC/1.2-20150622-intel-2017b
module load vsc-mympirun
export WORKDIR=$VSC SCRATCH/$PBS JOBID
mkdir -p $WORKDIR
cd $WORKDIR
# Test data
cp /apps/gent/tutorials/machine learning/classification/Indian/indian processed test.el .
# Model data
cp /user/home/gent/vsc425/vsc42544/indian processed training.el.model .
# by default, mympirun will use all available cores
# use --hybrid to only use a certain number of cores (per workernode)
# mympirun --hybrid 6 dbscan -e 300 -m 100 -t 12 bremenSmall.h5.h5
mympirun --hybrid 32 pisvm-predict indian processed test.el indian processed training.el.model results.txt
echo "Results available in $WORKDIR"
```

[Video] Maximum Margin



[7] YouTube Video, Text Classification 2: Maximum Margin Hyperplane'

Lecture Bibliography



Lecture Bibliography

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