FACULTEIT WETENSCHAPPEN

CENTRUM VOOR X-STRALENTOMOGRAFIE

# MECHANICA HOOFDSTUK 11

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UNIVERSITEIT GENT

## IMPULSMOMENT, ALGEMENE ROTATIE

## IMPULSMOMENT

- Analogieën met translatie-beweging
- Wat met vectoren en impuls?

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## IMPULSMOMENT

- Analogie

$$p = mv$$

$$\sum \tau = I\alpha$$

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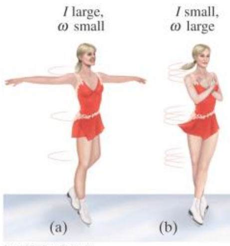
## IMPULSMOMENT

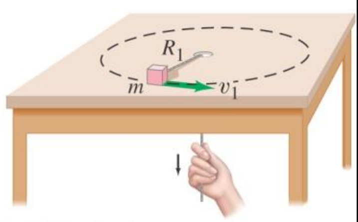
*Systeem zonder netto uitwendig krachtmoment:*




## BEHOUD VAN IMPULSMOMENT

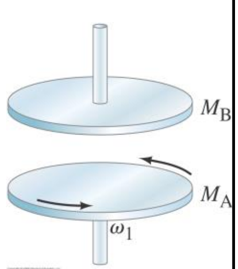
- Wat gebeurt er wanneer straal verkleint?
- Geen netto krachtmoment (waarom?)

$$I_1\omega_1 = I_2\omega_2$$




## BEHOUD VAN IMPULSMOMENT

- $M_A = 6{,}0\,\mathrm{kg}$; $M_B = 9{,}0\,\mathrm{kg}$
- $R_0 = 0{,}60\,\mathrm{m}$
- $\omega_1 = 7{,}2\,\mathrm{rad/s}$ in $\Delta t = 2{,}0\,\mathrm{s}$
- a) $L_A$; b) $\tau$; c) $\omega_2$ na koppeling



## BEHOUD IMPULSMOMENT

- Neutronenster: ster brandt op en valt samen tot neutronenster $(R \approx 10\,\mathrm{km})$

$$I_1\omega_1 = I_2\omega_2$$
$$\tfrac{2}{5}MR_1^2\omega_1 = \tfrac{2}{5}MR_2^2\omega_2$$
$$\Rightarrow f_2 = \frac{\omega_2}{2\pi} \approx 600\,\mathrm{Hz}$$

*Zgn. "ms-pulsars"*

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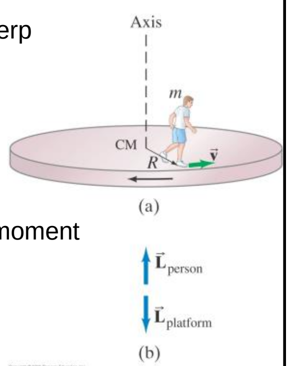
## VECTORIEEL IMPULSMOMENT

- Symmetrisch voorwerp om symmetrie-as

$$\bar{\mathbf{L}} = I\bar{\omega}$$

- Rechterhandregel!
- Behoud van impulsmoment

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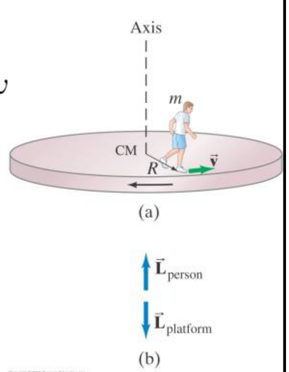
## VECTORIEEL IMPULSMOMENT

$$0 = mR^2\left(\frac{v}{R}\right) - I\omega$$

$$\omega = \frac{mRv}{I}$$

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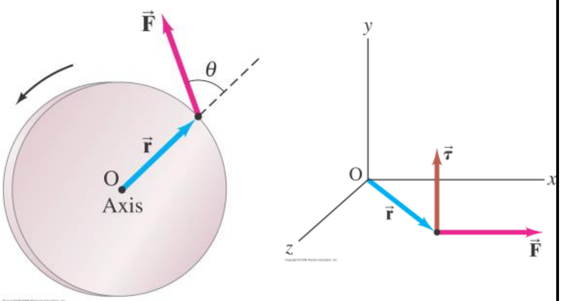
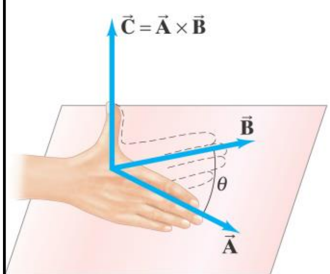
## KRACHTMOMENT VECTORIEEL

- Uitwendig vectorproduct! $\bar{\tau} = \bar{\mathbf{r}} \times \bar{\mathbf{F}}$




## UITWENDIG PRODUCT



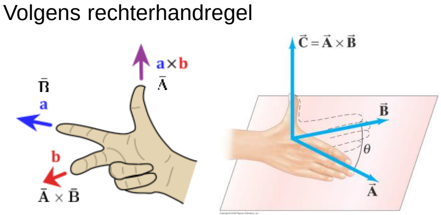

$$\bar{\mathbf{C}} = \bar{\mathbf{A}} \times \bar{\mathbf{B}}$$

*Vector!*

## UITWENDIG PRODUCT

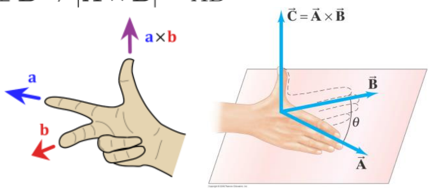
- Uitwendig product, vectorproduct
- Grootte $C = |\bar{\mathbf{A}} \times \bar{\mathbf{B}}| = AB\sin\theta$
- Richting
  - loodrecht op vlak $\bar{\mathbf{A}}, \bar{\mathbf{B}}$
  - Volgens rechterhandregel

*Rechts vs links = leerstof 2e leerjaar!*



## UITWENDIG PRODUCT

- $\bar{\mathbf{A}} \times \bar{\mathbf{A}} = 0$
- $\bar{\mathbf{A}} \times \bar{\mathbf{B}} = -\bar{\mathbf{B}} \times \bar{\mathbf{A}}$
- $\bar{\mathbf{A}} \times (\bar{\mathbf{B}} + \bar{\mathbf{C}}) = (\bar{\mathbf{A}} \times \bar{\mathbf{B}}) + (\bar{\mathbf{A}} \times \bar{\mathbf{C}})$
- $\bar{\mathbf{A}} \perp \bar{\mathbf{B}} \Rightarrow |\bar{\mathbf{A}} \times \bar{\mathbf{B}}| = AB$

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## UITWENDIG PRODUCT

$$\bar{\mathbf{A}} \times \bar{\mathbf{B}} = \begin{vmatrix} \bar{\mathbf{e}}_x & \bar{\mathbf{e}}_y & \bar{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\bar{\mathbf{e}}_x + (A_z B_x - AxB_z)\bar{\mathbf{e}}_y + (A_x B_y - A_y B_x)\bar{\mathbf{e}}_z$$

In the case of a 2 × 2 matrix the determinant may be defined as:

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Similarly, for a 3 × 3 matrix $A$, its determinant is:

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} \square & \square & \square \\ \square & e & f \\ \square & h & i \end{vmatrix} - b\begin{vmatrix} \square & \square & \square \\ d & \square & f \\ g & \square & i \end{vmatrix} + c\begin{vmatrix} \square & \square & \square \\ d & e & \square \\ g & h & \square \end{vmatrix}$$

$$= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= aei + bfg + cdh - ceg - bdi - afh.$$

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## IMPULSMOMENT VECTORIEEL

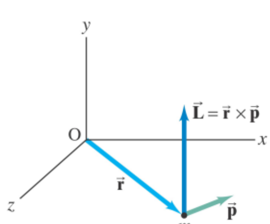
- Puntmassa
- Bij zuivere rotatie
- Definiëren: $\bar{\mathbf{L}} = \bar{\mathbf{r}} \times \bar{\mathbf{p}}$

$L = |\bar{\mathbf{L}}| =$

$=$

$=$

$=$

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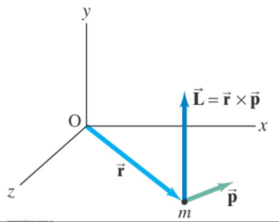
## IMPULSMOMENT VECTORIEEL

– Impulsmoment van een puntmassa:

$$\bar{\mathbf{L}} = \bar{\mathbf{r}} \times \bar{\mathbf{p}}$$

$$\frac{\mathrm{d}\bar{\mathbf{L}}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\bar{\mathbf{r}} \times \bar{\mathbf{p}}) = \frac{\mathrm{d}\bar{\mathbf{r}}}{\mathrm{d}t} \times \bar{\mathbf{p}} + \bar{\mathbf{r}} \times \frac{\mathrm{d}\bar{\mathbf{p}}}{\mathrm{d}t}$$




## IMPULSMOMENT VECTORIEEL

– Algemeen systeem

$$\frac{\mathrm{d}\bar{\mathbf{L}}}{\mathrm{d}t} = \sum_i \bar{\mathbf{r}} \times \bar{\mathbf{F}} = \sum_i \bar{\tau}_{\mathrm{uitw}}$$

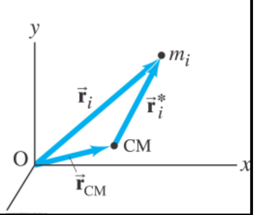
– T.o.v. zelfde oorsprong
– Enkel in inertiaalstelsel

## IMPULSMOMENT T.O.V. MM

– Ook geldig t.o.v. MM
– Ontbind elke positievector $\bar{\mathbf{r}}_i = \bar{\mathbf{r}}_{MM} + \bar{\mathbf{r}}_i^*$

$$\bar{\mathbf{p}}_i = m_i \frac{\mathrm{d}\bar{\mathbf{r}}_i}{\mathrm{d}t} = m_i \frac{\mathrm{d}(\bar{\mathbf{r}}_{MM} + \bar{\mathbf{r}}_i^*)}{\mathrm{d}t} = m_i \bar{\mathbf{v}}_{MM} + m_i \bar{\mathbf{v}}_i^* = m_i \bar{\mathbf{v}}_{MM} + \bar{\mathbf{p}}_i^*$$

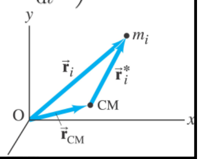
$$\bar{\mathbf{L}}_{MM} = \sum_i (\bar{\mathbf{r}}_i^* \times \bar{\mathbf{p}}_i^*)$$



## IMPULSMOMENT T.O.V. MM

$$\bar{\mathbf{L}}_{MM} = \sum_i (\bar{\mathbf{r}}_i^* \times \bar{\mathbf{p}}_i^*) \qquad \bar{\mathbf{p}}_i = m_i \bar{\mathbf{v}}_{MM} + \bar{\mathbf{p}}_i^*$$

$$\begin{aligned}
\frac{\mathrm{d}\bar{\mathbf{L}}_{MM}}{\mathrm{d}t} &= \sum_i \left(\frac{\mathrm{d}\bar{\mathbf{r}}_i^*}{\mathrm{d}t} \times \bar{\mathbf{p}}_i^*\right) + \sum_i \left(\bar{\mathbf{r}}_i^* \times \frac{\mathrm{d}\bar{\mathbf{p}}_i^*}{\mathrm{d}t}\right) \\
&= \sum_i \left(\bar{\mathbf{r}}_i^* \times \frac{(\mathrm{d}\bar{\mathbf{p}}_i - m_i \bar{\mathbf{v}}_{MM})}{\mathrm{d}t}\right) \\
&= \sum_i \left(\bar{\mathbf{r}}_i^* \times \frac{\mathrm{d}\bar{\mathbf{p}}_i}{\mathrm{d}t}\right) - \sum_i \left(m_i \bar{\mathbf{r}}_i^* \times \frac{\mathrm{d}\bar{\mathbf{v}}_{MM}}{\mathrm{d}t}\right) \\
&= \sum_i (\bar{\mathbf{r}}_i^* \times \bar{\mathbf{F}}_i) \\
&= \sum_i \bar{\tau}_i
\end{aligned}$$

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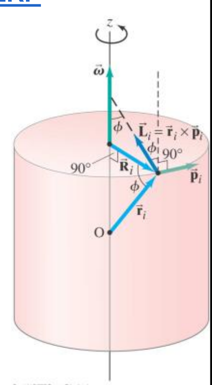
## STAR VOORWERP

- $\bar{\mathbf{L}} = \bar{\mathbf{r}} \times \bar{\mathbf{p}}$
- Ontbind in componenten langs rotatie-as en radiaal
- $\bar{\mathbf{L}} = L_\omega \bar{\mathbf{e}}_\omega + L_R \bar{\mathbf{e}}_R$
- Symmetrisch t.o.v. as:
  - Radiale componenten van impulsmomenten van punten aan overliggende zijde heffen elkaar op
  - $\bar{\mathbf{L}} = I\bar{\omega}$

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## MACHINE VAN ATWOOD

- Eerder in Hoofdstuk 4: katrol verwaarloosd

$$a = \frac{m_B - m_A}{m_A + m_B} g$$

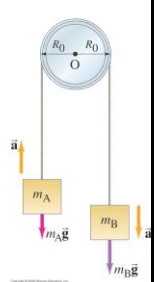
- Nu weten we dat $\tau = \frac{\mathrm{d}L}{\mathrm{d}t}$

$$m_B g R_0 - m_A g R_0 = \frac{\mathrm{d}L}{\mathrm{d}t}$$

$$\begin{aligned}
L &= m_A v R_0 + m_B v R_0 + I\omega \\
&= (m_A + m_B) R_0 v + I \frac{v}{R_0}
\end{aligned}$$

$$\bar{\mathbf{L}}_A = \bar{\mathbf{r}}_A \times \bar{\mathbf{p}}_A$$

$$\Rightarrow L_A = R_0 p_A$$

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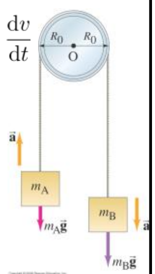
## MACHINE VAN ATWOOD

$$\tau = \frac{\mathrm{d}L}{\mathrm{d}t} \qquad L = (m_A + m_B)R_0 v + I\frac{v}{R_0}$$

$$(m_B - m_A)gR_0 = \frac{\mathrm{d}(m_A + m_B)R_0 v + I\frac{v}{R_0}}{\mathrm{d}t}$$

$$(m_B - m_A)gR_0 = \left((m_A + m_B)R_0 + \frac{I}{R_0}\right)\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\Rightarrow a = \frac{m_B - m_A}{m_A + m_B + \frac{I}{R_0^2}}g$$

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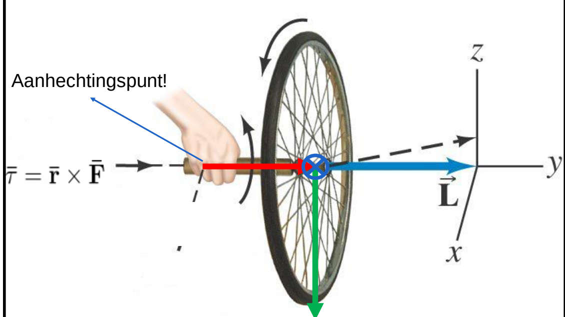
## FIETSWIEL



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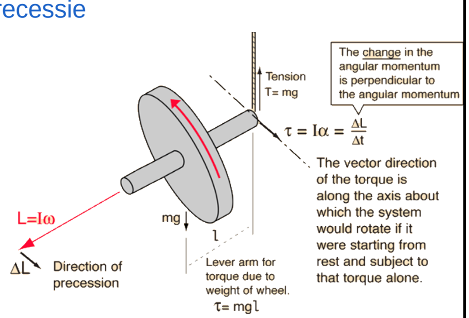
## FIETSWIEL

– Wiel zal omheen aanhechtingspunt draaien = precessie

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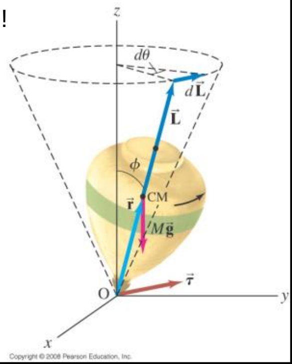
## TOL

– Precessiebeweging!

$$\mathrm{d}L = (L\sin\phi)\mathrm{d}\theta = |\tau_{\mathrm{net}}|\mathrm{d}t$$

$$\Rightarrow \Omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$$

$$= \frac{1}{\mathrm{d}t}\frac{\mathrm{d}L}{L\sin\phi}$$

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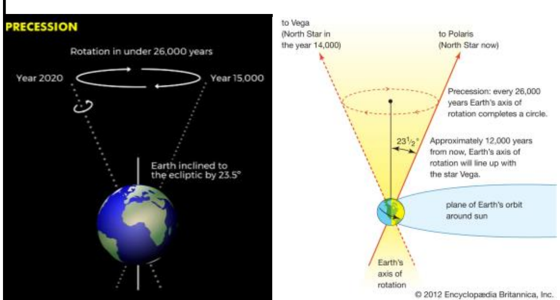
## PRECESSIE

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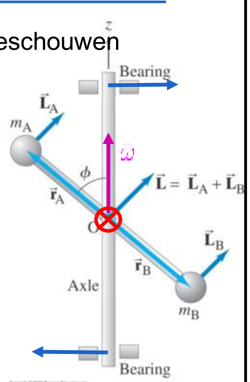
## NIET-GEBALANCEERDE ROTATIE

– Enkel massa's bollen beschouwen
– Wat is de rotatievector?
– Hoe verloopt het impulsmoment?

$$\frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \tau$$

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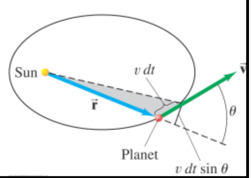
## 2E WET VAN KEPLER

- Herinner u de “wet der perken”
- 1 “perk” $\mathrm{d}A = \frac{1}{2}(r)(v\mathrm{d}t\sin\theta)$
- Impulsmoment planeet om zon

$$L = |\bar{\mathbf{r}} \times m\bar{\mathbf{v}}|$$
$$= mrv\sin\theta$$
$$= c^e$$

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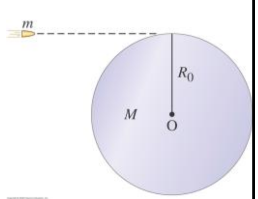
## KOGEL IN CILINDER

- Analoog aan ballistische slinger

$$L_i = L_e$$
$$R_0 mv = I\omega$$

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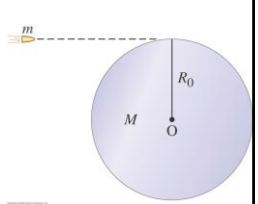
## KOGEL IN CILINDER

- Blijft kinetische energie behouden?

$$K_e - K_i =$$
$$=$$

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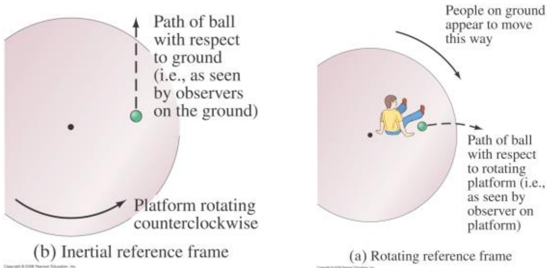
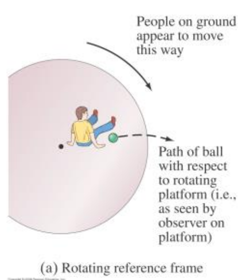
## (NIET-)INERTIAALSTELSELS

- In eerdere hoofdstukken:
  - Inertiaalselsel = niet versnellend stelsel
  - Tweede wet van Newton is geldig




## NIET-INERTIAALSTELSELS

- Hoe beschrijven we beweging vanuit niet-inertiaalselsels (bv. roterend vlak)?
- Fictieve krachten! (schijnkrachten)
  - Centifugaalkracht
  - Corioliskracht
- Deze krachten bestaan niet (er is niets dat ze veroorzaakt)

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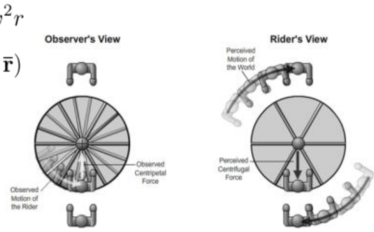
## CENTRIFUGAALKRACHT

- Cenripetaalkracht:
  - In vast stelsel nodig voor cirkelbaan
  - In roterend stelsel: tegen centrifugale kracht

$$F_{cf} = \frac{mv^2}{r} = m\omega^2 r$$

$$\bar{\mathbf{F}}_{cf} = -\bar{\omega} \times (\bar{\omega} \times \bar{\mathbf{r}})$$

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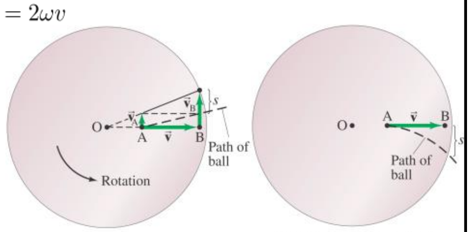
## CORIOLISKRACHT

– Vast stelsel: snelheid van A en B anders
$\bar{\mathbf{v}}_{\text{bal}} = \bar{\mathbf{v}} + \bar{\mathbf{v}}_A$

$$s = (r_B - r_A)\omega t = \omega v t^2 \qquad \bar{\mathbf{a}}_{\text{cor}} = -2\omega \times \bar{\mathbf{v}}$$

$$\Rightarrow a_{\text{cor}} = 2\omega v$$



(a) Inertial reference frame (b) Rotating reference frame

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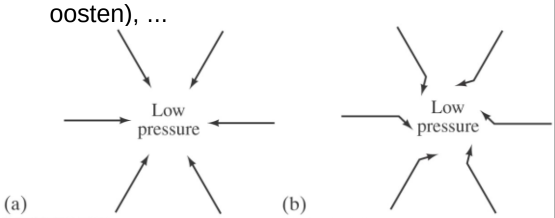
## CORIOLISKRACHT

– Op noordelijk halfrond: tegenijzerszin
– “Buys Ballot” wet
– Orkanen, stormen, oceaanstromingen, ...
– Ook ballistiek, vrije val (afwijking naar oosten), ...



(a) (b)

# EINDE HOOFDSTUK 11

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