De numerieke oplossing van het 3D Schrödinger-probleem

promotor: prof. M. Van Daele

Doelgroep: studenten met interesse in numerieke wiskunde

We zoeken de numerieke oplossing voor het drie-dimensionale Schrödinger probleem

$$-\nabla^2\psi + V(x,y,z)\psi = E\psi\,.$$

over (een deel van) $\mathbb{R}^3$. Aan de rand van dit gebied veronderstellen we dat $\psi(x,y,z) = 0$. In deze uitdrukking staat ∇ voor

$$\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\,.$$

De techniek die we zullen gebruiken om het probleem op te lossen, is een uitbreiding van een techniek die reeds uitgewerkt is voor het equivalente twee-dimensionale probleem.

Bij de gebruikte techniek dient de ∇-operator te worden gediscretiseerd in een grid van roosterpunten en in elk roosterpunt (x_i, y_j, z_k) wordt de oplossing ontwikkeld in een specifieke basis die afhangt van dit punt. Om het iets meer concreet te maken: we steunen onder andere op volgende ideeën

- in één dimensie kennen we bvb. de klassieke benaderingsformule van tweede orde:

  $$\frac{d^2 f(x)}{dx^2} = \frac{f(x+h) - 2\,f(x) + f(x-h)}{h^2}$$

  Deze benadering kan nog nauwkeuriger gemaakt worden door over te gaan op een vijf-, zeven-, ..., -puntsformule van orde 4, 6, ... Ook voor de functie ψ zullen we dergelijke benaderingen maken.

- een functie $f(x)$ van één reële veranderlijke x kan in vele basissen ontwikkeld worden: wiskundig is de meest eenvoudige keuze 1, x, x^2, ..., maar het is alom geweten dat dit numeriek een zeer instabiele basis is. Een basis op basis van orthogonale veeltermen (zoals bvb. Chebyshev) is een veel betere keuze. Maar in dit algoritme gaan we gebruik maken van een orthogonale basis van oplossingen van een gerelateerd één dimensionaal Schrödinger probleem. Deze basis is zeer geschikt omdat we via `Matslise` beschikken over goeie routines om deze basisfuncties te berekenen.

Wat wordt er verwacht bij deze masterproef? Er zijn, afhankelijk van de interesse van de student, meerdere mogelijkheden om zich op te focussen.

- theoretische aspecten bij het opstellen van de methode
- praktische aspecten bij de implementatie van de methode

Numerieke methoden voor tweede-orde differentiaalvergelijkingen

promotor: prof. M. Van Daele

Doelgroep: studenten met interesse in numerieke wiskunde

In de cursus Gevorderde numerieke methoden worden methoden besproken voor eerste orde differentiaalvergelijkingen. In principe volstaan deze om tweede-orde differtiaalvergelijkingen $y'' = f(x, y)$ op te lossen, want elke tweede-orde vergelijking kan omgezet worden in een stelsel eerste-orde vergelijkingen. Hierdoor wordt echter y' geïntroduceerd in het probleem. Om dit te vermijden, zijn er ook methoden ontwikkeld die rechtstreeks geschikt zijn voor tweede-orde (of algemeen gesproken hogere orde) differentiaalvergelijkingen. De meest bekende methode hiervoor is wellicht de Numerov-methode.

Het doel van deze thesis is na te gaan hoe deze methoden kunnen geconstrueerd worden. Wat is hun orde? Hoe zit de stabiliteitstheorie voor deze methoden in elkaar? ...

Literatuur

o.a : E. Hairer, S.P. Norsett, G. Wanner, Solving Ordinary Differential Equations I, Springer-Verlag (1991)

Exponential fitting

promotor: prof. M. Van Daele

Doelgroep: studenten met interesse in numerieke wiskunde

Heel veel numerieke methoden (voor de benadering van afgeleiden, de berekening van integralen, het oplossen van differentiaalvergelijken, ...) steunen op veelterminterpolatie. Voor sommige toepassingen (bvb. als geweten is dat het integrandum of de af te leiden functie periodiek is), zijn veeltermen niet zo geschikt, maar zijn trigonometrische of exponentiële functies eerder aangewezen. Daarom werden nieuwe methoden ontwikkeld die met dit oscillerend of exponentieel karakter rekening houden. Deze techniek heet exponential-fitting.

Het doel van deze thesis is dieper in te gaan op de exponential-fitting techniek en zo'n methoden toe te passen op een aantal typische problemen.

Literatuur

Liviu Gr. Ixaru and G. Vanden Berghe, Exponential Fitting, Kluwer, Boston - Dordrecht - London, 2004.
Present state-of-the-art in exponential fitting. A contribution dedicated to Liviu Ixaru on his 70th birthday, Computer Physics Communications 183 (2012) 2499-2512.

Een ander onderwerp in de numerieke wiskunde

promotor: prof. M. Van Daele

Doelgroep: studenten met interesse in numerieke wiskunde

Het onderwerp is bespreekbaar ...

Granular approximations: a new direction in fuzzy rough set theory

Promotor: Prof. Chris Cornelis
Begeleider: Marko Palangetić

Fuzzy Rough Sets (FRS) are a construct of artificial intelligence designed to deal with partially inconsistent information. For example, in a classification problem, two objects may be very similar in terms of their attribute values yet belong to different classes. In FRS theory, such inconsistencies are addressed by providing two approximations of a decision class C:

- The lower approximation of C contains objects certainly belonging to C
- The upper approximation of C contains objects possibly belonging to it

The lower and upper approximation can be interpreted as extreme cases of consistent sets, which limits their practical use. For this reason, recently the notion of a granular approximation was proposed: it is defined as the consistent fuzzy set which is closest to the original decision class C with respect to a given statistical loss function. Granular approximations can be obtained in practice by solving an optimization problem (in many cases, a linear program).

The goal of this thesis is to evaluate the potential of granular approximations in classification problems. First steps have already been taken to design an instance-based classifier, so the student can build on this work.

Depending on the student's interests, the performed research can entail a mix of more fundamental aspects (investigating theoretical properties of the approximations, in function of the used fuzzy logic connectives) and applied ones (setting up an experimental study to tune the algorithm's hyperparameters and comparing it with state-of-the-art classifiers).

As this is cutting-edge research, there is a good chance that it will lead to an academic research publication.

References

[1] M. Palangetić, C. Cornelis, S. Greco, R. S{\l}owinski, Granular approximations: A novel statistical learning approach for handling data inconsistency with respect to a fuzzy relation, Information Sciences 629, p. 249-275, 2023
[2] M. Palangetić, C. Cornelis, S. Greco, R. S{\l}owinski, Multi-class granular approximation by means of disjoint and adjacent fuzzy granules, Fuzzy Sets and Systems 478, 2024

Entropy of covering based rough sets

Promotor: Prof. Chris Cornelis
Begeleider: Wannes De Maeyer

Rough sets where introduced by Pawlak to deal with inconsistent data using an equivalence relation that links indiscernible data [1]. Covering-based rough sets (CBRS) are an extension of classical rough sets that replace the equivalence relation with a covering [2]. CBRS have been successfully used in many machine learning applications, including feature selection and classification.
When using CBRS in an application, selecting the right approximation operator and corresponding covering is important. Because of this, multiple measures of approximation operators have been introduced. One such measure is entropy [3], but calculating the entropy is computationally quite expensive. Because of this, estimations of entropy need to be made. Calculating these estimations is algorithmically complex and interesting.

The goal of this thesis is to estimate the entropy of various covering-based rough set approximation operators and to use this measure to find appropriate coverings for applications in machine learning. Depending on the interests of the student this can be purely theoretical, where we would mostly search for theoretical results of the entropy of CBRS approximation operators and good estimations of their entropy. Alternatively, we can could look at possible applications of entropy, and study the relation of the entropy of an approximation operator to the results of machine learning algorithms using this operator. The research questions are open for discussion.

References:

[1] Z. Pawlak, Rough Sets, International Journal of Computer and Information Sciences 11(5), pp. 341-356, 1982.
[2] Yao, Y., & Yao, B. (2012). Covering based rough set approximations. *Information Sciences*, *200*, 91–107. doi:10.1016/j.ins.2012.02.065
[3] Zhu, P., & Wen, Q. (2012). Entropy and co-entropy of a covering approximation space. *International Journal of Approximate Reasoning: Official Publication of the North American Fuzzy Information Processing Society*, *53*(4), 528–540. doi:10.1016/j.ijar.2011.12.004

Similarity learning for fuzzy rough rule induction

Promotor: Prof. Chris Cornelis
Begeleider: Henri Bollaert

Fuzzy Rough Sets (FRS) are a construct from artificial intelligence that has been successfully applied in various machine learning tasks, including feature selection, instance selection, classification, and regression [1].

FRS make use of a similarity relation, which expresses the degree to which two elements in a dataset are related. Instead of using a normal, fixed similarity relation, it is possible to apply similarity learning or distance metric learning (DML)[2] to learn the optimal relation from the data. This has been successfully applied to neighborhoud-based machine learning methods, such as k-nearest neighbours.

Rule induction is a machine learning model which learns rules from the data with the aim of e.g. predicting the class of new samples. One such technique which makes use of FRS theory, is QuickRules [3], which creates rules in a greedy way.

The goal of this thesis is to design and conduct an extensive experimental study on a collection of benchmark datasets, in order to evaluate the predictive and descriptive potential of similarity learning in the context of rule induction.

The specific research questions that the thesis should address are:

• How can we apply DML to rule induction?

• Which DML methods perform best in this context, what are their strengths and weaknesses w.r.t. predictive and descriptive performance?

• Can we improve these methods or create our own technique which outperforms them?

• Are there specific kinds of data (e.g., imbalanced data, high-dimensional data, multi-label data, numerical vs categorical data, ...) on which the approach performs better or worse?

Prior knowledge of fuzzy rough sets is not necessary, and a Python implementations of QuickRules will be provided. On the other hand, experience with setting up machine learning experiments is highly recommended.

References

[1] S. Vluymans, et al. “Applications of Fuzzy Rough Set Theory in Machine Learning: a Survey.” Fundamentae Informaticae 142.1-4 (2015): 53-86.
[2] J. L. Suárez, S. García, F. Herrera, A tutorial on distance metric learning: Mathematical foundations, algorithms, experimental analysis, prospects and challenges, Neurocomputing 425 (2021) 300–322.
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GHENT UNIVERSITY

Master thesis

Analysis and Numerical Simulation of a new Shallow Water Model

Course of study: Mathematics or Applied Mathematics
Kind of thesis: Mathematical Modeling or Numerical Simulation or Scientific Computing

Problem

Free-surface flows model the evolution of water in oceans or rivers, e.g., the computation of a **tsunami** or a **dam break**. Numerical simulations often use the standard Shallow Water Equations (SWE), see figure 1. The SWE are easy to solve, but often inaccurate. The evolution equations for water height h and velocity u, read

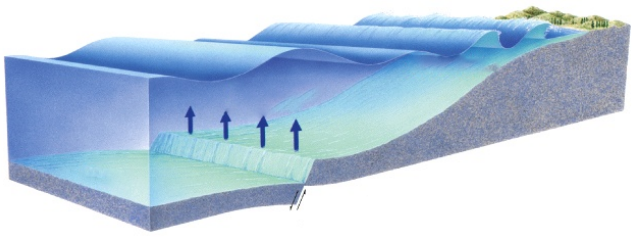

Figure 1: Shallow Water setup.

$$\partial_t h + \partial_x (hu) = 0, \quad \partial_t (hu) + \partial_x \left(hu^2 + \frac{1}{2} g h^2 \right) = P(u, h).$$

In this project, we want to compute more accurate solutions of tsunami simulations like in figure 2. We use a recently developed advanced Shallow Water model, called *Shallow Water Moment Equations* (SWME). This model extends the accuracy of existing models.

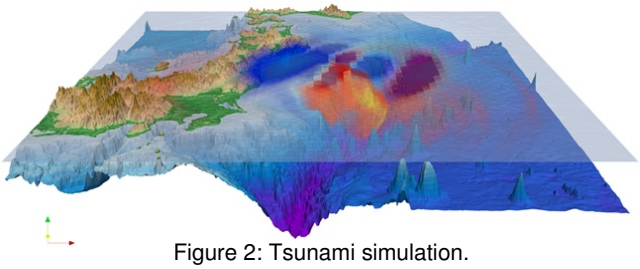

Figure 2: Tsunami simulation.

Goal

This project's goal is to use the advanced model to analyze and simulate more accurate free-surface flows. The detailed goal depends on the interests of the student and can vary between an analytical investigation of the model, a numerical simulation, or the extension of existing software tools for this model.

Preliminary work

The model has been proposed in 2019 and since then a number of publications has addressed 1D simulations, stability properties, steady-states, and the extension to 2D.

Tasks

The theoretical, applied or computational work can include (in part)

- Implementation of new numerical schemes for a faster solution of the model
- Comparison of the model with experimental data or other models
- Derivation and investigation of boundary conditions
- Increasing the accuracy of the model by further model reduction

The final tasks will be discussed with the supervisor. Please feel free to get in contact.

Supervision

Contact: **Prof. Dr. Julian Koellermeier**
Email: julian.koellermeier@ugent.be

GHENT UNIVERSITY

Master thesis

Analysis and Numerical Simulation for a new Shallow Water Model with Vegetation

Course of study: Mathematics

Problem

River flows are efficiently simulated using the shallow water equations. In the most simple version, the evolution equations for water height h and average velocity u, read

$$\partial_t h + \partial_x (hu) = 0, \quad \partial_t (hu) + \partial_x \left(hu^2 + \frac{1}{2} g h^2 \right) = -gh\partial_x b.$$

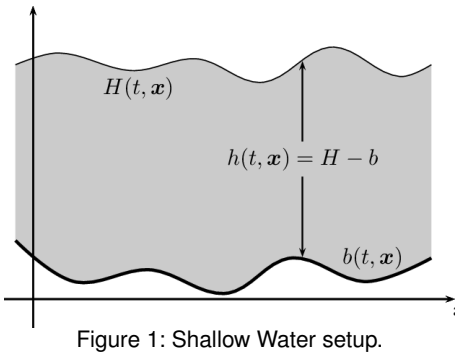

Figure 1: Shallow Water setup.

Recently, more advanced models have been developed that take into account more physical velocity profiles, for example slower velocities at the bottom of the river due to friction.
In this project, we want to study the effect of vegetation on the water flow. This requires more detailed knowledge about the velocity profile and its interaction with the vegetation. Most commonly, vegetation is modeled by an additional friction term in the model. This friction term then reduces the velocity only in the regions of vegetation. Different scenarios include, e.g., submerged or floating vegetation.

Goal

This project's goal is to use newly developed models to analyze and simulate more accurate river flow with vegetation. The detailed goal depends on the interests of the student and can vary between an analytical investigation of the model, a numerical simulation, or the extension of existing software tools for this model.

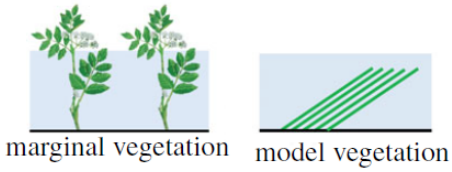

Figure 2: Vegetation in water.

Preliminary work

The model has been proposed in 2019 and was followed by 1D simulations, stability properties, steady-states analysis, and the extension to 2D. The inclusion of vegetation was explored in a student project. This work builds up on that..

Tasks

The theoretical, applied or computational work can include (in part)

- Addition of a vegetation model to the extended shallow water equations
- Analysis of the coupled model
- Inclusion of a vegetation growth model
- Comparison of the model with experimental data or other models

The final tasks will be discussed with the supervisors. Please feel free to get in contact.

Supervision

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Eindige integratiemethode gebaseerd op Feynman-Kac formule voor het prijzen van opties met meerdere onderliggenden
Promotor: Prof. dr. M. Vanmaele
Doelgroep: studenten uit de tweede master Wiskunde

Korte beschrijving

Het onderwerp van deze masterproef is de studie van een onvoorwaardelijk stabiele numerieke methode om de prijs van een optie met hoge nauwkeurigheid te bepalen als oplossing van een meerdimensionale Black-Scholes-vergelijking, zie Cho et al. (2023). Het voorgestelde schema gebruikt de operator-splitsingsmethode om de meerdimensionale partiële differentiaal-vergelijking te reduceren tot een verzameling van eendimensionale subproblemen. Het berekeningsdomein wordt gediscretiseerd met een uniforme ruimte- en tijdstapgrootte om de optiewaarden en correlatiegerelateerde termen te benaderen met stuksgewijze kwadratische veeltermen. Daarnaast wordt via Feynman-Kac de oplossing geschreven als een conditionele verwachtingswaarde. Aangezien in een Black-Scholes setting de prijs van een onderliggende lognormaalverdeeld is, kan de numerieke oplossing van de deelproblemen verkregen worden door de veeltermen analytisch te integreren om deze verwachtingswaarden te berekenen. De methode wordt vergeleken met de impliciete operator-splitsingsmethode (OSM), een veelgebruikte eindige differentiemethode. Uit numerieke experimenten zou moeten blijken dat de voorgestelde methode beter presteert dan OSM in termen van convergentie in ruimte- en tijdrichtingen. Daarnaast wordt er ook verwacht een analyse uit te voeren die de onvoorwaardelijke stabiliteit van deze methode garandeert.
Essentieel bij deze methode is dat de (transitie)dichtheidsfunctie van de prijs van de onderliggende bekend is. Eens vertrouwd met de methode, zou er kunnen nagegaan worden of deze methode kan uitgebreid worden naar opties waarbij de prijs van de onderliggende een ander proces volgt waarvoor de transitiedichtheid analytisch bekend is, zoals bijvoorbeeld een Ornstein-Uhlenbeck-proces. Een andere vraag die kan onderzocht worden is: “Is er een verband met de eindige integratiemethode in de artikels van Li et al. (2015) en van Ma et al. (2024)?”

Artikels:

- Junhyun Cho, Donghee Yang, Yejin Kim, Sungchul Lee, An operator splitting method for multi-asset options with the Feynman-Kac formula. Computers & Mathematics with Applications, Volume 135, 2023, Pages 93-101, https://doi.org/10.1016/j.camwa.2023.01.019
- Li M, Chen CS, Hon YC, Wen PH., Finite integration method for solving multi-dimensional partial differential equations. Applied Mathematical Model 2015a; 39(17):4979–94. https://doi.org/10.1016/j.apm.2015.03.049
- Y. Ma, C.Z. Shi, Y.C. Hon, Generalized Finite Integration Method with Laplace transform for European option pricing under Black–Scholes and Heston models. Engineering Analysis with Boundary Elements, Volume 164, 2024,105751, https://doi.org/10.1016/j.enganabound.2024.105751

Pricing swing option with neural network
Promotor: Prof. dr. M. Vanmaele
Begeleider: Mustapha Regragui
Doelgroep: studenten uit de tweede master Wiskunde

Korte beschrijving

After the deregulation of the energy market, various derivative products became popular, especially swing options. This latter is among the most traded options in the electricity market. These options, characterized by their flexible exercise features coupled with some constraints to reduce the exposure to strong price fluctuations.
Swing option valuation has relied on numerical techniques such as the Monte Carlo simulation method or the numerical solution of partial integro-differential equations (PIDEs). However, with the rise of deep learning, neural networks have demonstrated substantial promise in various fields, particularly in the approximation of complex functions and high-dimensional representations. Recently, these tools have been applied to financial engineering problems, including the valuation of complex derivatives product such as swing options, see Deschatre at al. (2022) and Lemaire et al. (2024) .
This thesis aims to conduct an in-depth investigation into the robustness and accuracy of neural network-based approaches for the valuation of swing options. The study will encompass also different stochastic models for the underlying electricity price.

Relevante literatuur vormen de volgende artikels:

- Vincent Lemaire, Gilles Pagès, Christian Yeo, A new Input Convex Neural Network with application to options pricing, 2024. https://doi.org/10.48550/arXiv.2411.12854
- Vincent Lemaire, Gilles Pagès, Christian Yeo, Swing contract pricing: With and without neural networks. Frontiers of Mathematical Finance, 2024, 3(2): 270-303. https://doi.org/10.3934/fmf.2024007
- Thomas Deschatre, Joseph Mikael, Deep combinatorial optimisation for optimal stopping time problems: application to swing options pricing. MathematicS In Action, Special issue Maths and Industry, Volume 11 (2022) no. 1, pp. 243-258. https://doi.org/10.5802/msia.26

Electricity price model calibration
Promotor: Prof. dr. M. Vanmaele
Begeleider: Mustapha Regragui
Doelgroep: studenten uit de tweede master Wiskunde

Korte beschrijving
Electricity prices exhibit unique and complex dynamics that set them apart from traditional financial assets. Unlike commodities, electricity - at least for big volumes- cannot be stored leading to

distinctive behaviour such as high volatility, presence of frequent spikes, mean-reversion tendency and daily, weekly and annually seasonal patterns. These features make the modelling and valuation of electricity derivatives, especially swing options, a challenging task.
Various stochastic models have been developed to capture these features of electricity prices. However, one of the challenges lies in accurately calibrating these models to reflect real market behaviour.
This thesis aims to investigate the calibration of several stochastic electricity price models and to determine which model best aligns with actual market data.

Voor deze studie kan gesteund worden op de volgende artikels:

- Martin Barlow, Yuri Gusev, Manpo Lai, Calibration of multifactor models in electricity markets. International Journal of Theoretical and Applied Finance, 2004; 07(02):101-120. https://doi.org/10.1142/S0219024904002396
- Emanuele Fabbiani, Andrea Marziali, Giuseppe De Nicolao, Fast calibration of two-factor models for energy option pricing. Applied Stochastic Models in Business and Industry, 2021; 37: 661–671.
  https://doi.org/10.1002/asmb.2604

Een ander onderwerp in het domein van de financiële of actuariële wiskunde
Promotor: Prof. dr. M. Vanmaele
Doelgroep: studenten uit de tweede master Wiskunde

Bespreekbaar

ARITHMETIC OF QUATERNION ALGEBRAS AND SUPERSINGULAR ELLIPTIC CURVES

LAM PHAM

Overview

Quaternion algebras sit at the crossroads of algebraic number theory, the arithmetic of algebraic groups, and the geometry of modular curves. They provide a unifying language for several deep results—local–global principles such as the *Hasse norm theorem*, explicit formulas for *class and type numbers*, various *mass formulas*, and the computation of *Tamagawa numbers*. A celebrated application is the **Deuring correspondence**, which gives a bijection between isomorphism classes of supersingular elliptic curves over $\overline{\mathbb{F}}_p$ and ideal classes in a maximal order of a definite quaternion algebra over $\mathbb{Q}$. This has important applications to post-quantum cryptography.

Aims

The thesis divides naturally into two parts.

(1) **Foundational part.** Develop a solid understanding of quaternion algebras over number fields, their local invariants, and the adelic perspective. Milestones include:
   - proofs and applications of the Hasse norm theorem;
   - Formulas for class and type numbers of quaternion orders;
   - Eichler, Tamagawa and Harder mass formulas interpreted via volumes of adelic groups;
   - computation and meaning of Tamagawa numbers for $\mathrm{SL}_1(B)$.

(2) **Application part.** Derive and analyse the Deuring correspondence:
   - construct the bijection between supersingular elliptic curves (up to isomorphism) and ideal classes of a maximal order in the definite quaternion algebra ramified exactly at p and ∞;
   - explore computational aspects (Brandt matrices, algorithms of Pizer and Kohel);

Possible Project Trajectories

- **Local–to–global methods.** Trace how local invariants control global structure; compare quaternionic results with those for quadratic forms.
- **Bruhat–Tits trees and maximal orders.** For a quaternion algebra over a p–adic field, realise maximal orders as vertices of the Bruhat–Tits tree of GL_2; study the action of $\mathrm{SL}_1(B)$ on the tree and relate geodesics to optimal embeddings and supersingular isogeny graphs.
- **Volume formulas and automorphic forms.** View the mass formula as a volume computation in adelic $\mathrm{SL}_1(B)$ and relate it to Tamagawa numbers and automorphic representations.
- **Historical perspective.** Follow the development from Hasse's local classification, through Deuring's work on elliptic curves, to modern treatments via algebraic groups and adèles.

Core References

(1) M.–F. Vignéras, *Arithmétique des algèbres de quaternions*, LNM 800, Springer (1980).
(2) M. Eichler, *Quadratische Formen und orthogonale Gruppen*, Springer (1952).
(3) M. Deuring, "Die Typen der Multiplikatorenringe elliptischer Funktionenkörper," *Abh. Math. Sem. Hamburg* 14 (1941), 197–272.
(4) A. Pizer, "An algorithm for computing modular forms on $\Gamma_0(N)$," *J. Algebra* 64 (1980), 340–390.
(5) H. Tamagawa, "On the ζ–function of a division algebra," *Ann. of Math.* 77 (1963), 127–151.

WEIL'S CLASSIFICATION OF CLASSICAL GROUPS VIA ALGEBRAS WITH INVOLUTION

LAM PHAM

André Weil showed that every classical linear algebraic group over a field K – the general, special and projective linear, orthogonal, symplectic and unitary groups—can be realised as the K-rational automorphism group of a suitable *central simple K-algebra equipped with an involution*. This perspective unifies the theories of central simple algebras, quadratic and Hermitian forms, and Galois cohomology, and provides a powerful framework for classifying K-forms of classical groups. As an application, this gives a classification of arithmetic subgroups of classical groups.

The thesis is structured around two main objectives.

(1) **Foundational development.** Build a strong theoretical foundation in the following topics:
- Central simple algebras (CSA): Wedderburn's theorem, the Brauer group $\mathrm{Br}(K)$, splitting fields.
- Involutions on CSAs: orthogonal, symplectic and unitary involutions; examples via matrix algebras.
- Quadratic, symplectic and Hermitian forms: Witt decomposition, isotropy criteria over local and global fields.
- Essentials of non-abelian Galois cohomology: torsors and the set $H^1(K, G)$.

(2) **Weil's classification.** Prove and explore Weil's theorem:
- Construct $\mathrm{U}(A, \sigma) = \{x \in A^\times \mid \sigma(x)x = 1\}$ and its special/projective variants.
- Apply the Skolem–Noether theorem to classify involutions of the first kind on $M_n(K)$.
- Show how each classical group arises as $\mathrm{U}(A, \sigma)$ for a suitable (A, σ).
- Translate isomorphism classes of groups into invariants in the Brauer group and Galois cohomology.

References.

(1) A. Weil, *Algebras with involutions and the classical groups*, J. Indian Math. Soc., n. Ser. 24, 589-623 (1961).
(2) J.-P. Serre, *Galois Cohomology*, Springer (1997).
(3) I. Scharlau, *Quadratic and Hermitian Forms*, Springer (1985).
(4) A. Borel, *Introduction to arithmetic groups* AMS University Lecture Series (2019)
(5) D.-W. Morris, *Introduction to arithmetic groups*, Deductive Press (2015).

Acties op bomen: lokale-actie-diagrammen

Promotor: Tom De Medts

Het bestuderen van groepen die werken op (oneindige) bomen is een heel rijk onderzoeksonderwerp. Deze groepen hebben een natuurlijke topologie, die er totaal onsamenhangende lokaal compacte groepen (kortweg TDLC-groepen) van maakt.

Het is een natuurlijke vraag in hoeverre de "lokale structuur" van een dergelijke groep bepaalt wat de globale structuur is. Deze vraag is bijzonder interessant voor gesloten deelgroepen van automorfismengroepen van bomen die voldoen aan de befaamde onafhankelijkheidseigenschap (P) ingevoerd door Jacques Tits.

Zeer recent hebben Colin Reid en Simon Smith een theorie ontwikkeld om alle gesloten deelgroepen met eigenschap (P) te beschrijven, gebruik makend van wat ze *lokale-actie-diagrammen* noemen.

De bedoeling van deze thesis is vooreerst om deze theorie te bestuderen. Afhankelijk van de interesse van de student kan het nadien verschillende kanten uitgaan: de focus kan liggen op andere *local-to-global* resultaten (zoals bv. een diepere studie van het werk van Marc Burger en Shahar Mozes), op varianten van eigenschap (P), op veralgemeningen naar andere meetkundige structuren (zoals rechthoekige gebouwen), of op een algemene theoretische studie van TDLC-groepen.

Referenties

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Het Bimonster

Promotor: Tom De Medts

Het Monster $\mathbb{M}$ is de grootste van de 26 sporadische eindige enkelvoudige groepen. Deze groep expliciet beschrijven is niet eenvoudig. Deze groep werd voor het eerst expliciet geconstrueerd door Robert Griess als de automorfismengroep van een 196,884-dimensionale algebra.

Wat minder gekend is, is het feit dat het *Bimonster* $(\mathbb{M} \times \mathbb{M}) \rtimes C_2$ eigenlijk in sommige situaties natuurlijker is dan het Monster zelf. Deze groep heeft immers een vrij eenvoudige expliciete presentatie, die verkregen kan worden vanuit een Coxetergroep met een Y-diagram door het opleggen van één bijkomende relatie, de zogenaamde *spider-relatie*. Nog een andere manier om het Bimonster te beschrijven is door te starten van de Coxetergroep bepaald door het projectief vlak van orde 3 en een ander soort bijkomende relatie(s).

De bedoeling van deze thesis is om deze groep te bestuderen. Uiteraard gaat dit gepaard met het bestuderen van heel wat gerelateerde groepen, zoals algemene Coxetergroepen en andere eindige groepen die verkregen kunnen worden vanuit oneindige Coxetergroepen door het opleggen van één bijkomende relatie.

Het startpunt van de thesis is het bestuderen van het werk van Simon Norton, Leonard Soicher en Alexander Ivanov. Afhankelijk van de interesses van de student kan dit gepaard gaan met eigen computerimplementaties of met een theoretischere studie, of een zoektocht naar andere algbraïsche structuren waarin het Bimonster natuurlijk opduikt.

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Titel:Statistische besluitvorming na gebruik van machine learning technieken

Promotor: Prof. Stijn Vansteelandt

Korte beschrijving:

Meer en meer statistische analyses maken gebruik van machine learning technieken om bijvoorbeeld het effectvan een behandeling of interventie te schatten. Zoals alle statistische analyses, genereren ook machine learning technieken onzekerheid op de resultaten (ten gevolge van steekproefvariatie); omwille van de complexiteit van de technieken wordt dit in de verdere analyse echter meestal genegeerd. Als gevolg hiervan zijn de betrouwbaarheidsintervallen die men bekomt, typisch zwaarvertekend. In deze masterproef zult u betrokken worden in recente ontwikkelingen, alsook onderzoek binnen de onderzoeksgroep van de promotor, om na te gaan hoe men hiermee kan omgaan. De bestudeerde methodes zullen theoretische en/of door middelvan simulatiestudies en concrete data-analyses worden geëvalueerd. Er kan gekozen worden voor hetzij een meer theoretisch onderwerp (bvb., een studie van auto-dml) of betrokkenheid in lopend onderzoek (ter bespreking), of een meer toegepast onderwerp (bvb., gebruik van debiased machine learning voor de analyse van intensieve zorgen data of geneesmiddelenstudies; evaluatie door middelvan simulatie van de impact van de keuze van algoritmes in stacked learners,van het aantal folds in cross-fitting, ...; ontwikkelen van een R-pakket voor assumption-lean regression; ...).

Privacy-Preserving Causal Inference

Promoter: Máté Kormos

Causal inference aims at learning the effect of a treatment on an outcome, based on a random sample from the population of interest. For instance, the effect of a medical treatment on the well-being of individuals. We may distinguish between two scenarios: whether the treatment was administered randomly in an experiment (e.g. medical treatments) or individuals were free to make their own choice (e.g. obtaining a master's degree). In the latter case, individuals who decide to take the treatment may systematically differ from those who do not do so, which calls for different causal inference methods than the experimental scenario. For concreteness, some of these methods are regression adjustment, matching, augmented inverse propensity score weighting, instrumental variables.

Privacy protection aims at protecting the data of individuals in the sample by not disclosing them (for inference purposes). Broadly, two paradigms may be distinguished: when the data are protected by cryptographical methods (e.g. homomorphic encryption) or by simply injecting noise into them (e.g. differential privacy). The latter, although offers weaker privacy, is more suitable for inference purposes.

While causal and privacy-preserving inference have each been extensively studied, their combination has remained relatively unexplored. The aim of this topic is therefore to explore privacy-preserving causal inference. The concrete research question, which may range from applications to methodological developments, is to be decided depending on the student's preference.